Phase Transition Behavior of Message Propagation in Delay Tolerant Vehicular Ad Hoc Networks*

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Abstract–

The vehicular networking environment is characterized by time-varying traffic density and relatively high vehicular mobility rates. Enabling networking in the vehicular environment is challenging due to the rapidly changing topology and the vast network of roadways. In this article, we present an analytical model to describe the behavior of message propagation in a delay tolerant network formed over moving vehicles. We describe a model where partitioning between connected subnets exists in a network with time-varying topology. A messaging scheme exploits the time-varying partitioning to enable data exchange in a delay tolerant network setting. We derive the bounds for performance of message propagation in the vehicular networking environment. The analytical bounds derived are compared with simulation results for characteristics of the vehicular networking environment such as vehicular traffic density, transmission range and vehicular speed. The results depict the observation of phase transition in message propagation rate with increasing vehicle traffic density. Importantly, we are able demonstrate the limits of densities of bi-directional vehicle traffic at which the transitions occur. The results show that the delay tolerant networking assumption provides gains for message propagation over traditional ad hoc networking schemes that rely on path formation. Finally, we show that increased mobility of vehicles actually aids in message propagation, contrary to the expectation that it would be a hindrance due to frequent topology changes.

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1 Introduction

Vehicles will soon become an integral part of networked information and control systems [1, 2]. Vehicle manufacturers seek to enhance the vehicle ownership experience by enhancing safety and providing additional features such as Internet connectivity. Creating safer transportation system is an important goal. One way of achieving safety is by enabling inter-vehicle communication. Communication between vehicles allows sharing state information that is used to avoid contention in the system for shared resources and avoid accidents. By interconnecting vehicles with networking technology we will enable distributed local control applications such as accident prevention, route optimization, and traffic management; while permitting global optimizations that balance societal constraints such as throughput, regional energy use and air quality.

IEEE 802.11p – also called WAVE (Wireless Access in Vehicular Environments) provides the required standards and protocols to enable vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communication. The standard is a part of the DSRC (Dedicated Short Range Communication) spectrum allocated in the US. The primary goal is to enable public safety applications that can save lives and improve traffic flow. Similar efforts are initiated in Europe and Japan. Several commercial solutions that enable Internet access in vehicles exist.

While the standards and technologies enabling inter-vehicle communication are in development, we adopt a broader view of the vehicular networking environment and analyze messaging in a sparse density model. As the topology of a network formed over moving vehicles is highly variable, the connectivity of the network, absent any infrastructure, is largely dependant on the vehicular traffic density. Low traffic densities and the vast expanse of the road networks leads to a model that is highly partitioned. We consider a scenario where the vehicular networking environment is characterized by disconnected sub-nets on the roadway. The partitioning between the sub-nets is time-varying as vehicles traverse in opposing directions on a bi-directional roadway. A messaging scheme described in previous work [3] is able to exploit the time varying partitioning to enable message propagation between otherwise disconnected sub-nets.

In this paper, we discuss the phase transition phenomenon observed in message propagation in the described vehicular networking scenario. Importantly, we are able to derive the limiting condition for phase transition, a quantity that is not evident from the simulation results. We consider an infrastructure-less ad hoc network formed over moving vehicles in a highway scenario. We present an analytical model that qualitatively captures the behavior of message propagation rate in a fragmented network under the assumption of delay tolerant networking. The model is used to derive an upper bound and a lower bound on the message propagation rate as a function of vehicular traffic density, radio characteristics and vehicular speed. Results show the message propagation rate as the vehicular traffic density increases.

The rest of the paper is organized as follows – Section 2 describes related work. Section 3 details the vehicular networking environment and the relevant observations. We present a detailed description of our analytical model and derive the bounds on message propagation in Section 4. The simulation results are compared with the analytical model in Section 5 and finally, we conclude the paper in Section 6 with a discussion of the results.

2 Related Work

Consortia such as PATH (The Partners for Advanced Transit and Highway), C2CC (Car-To-Car Consortium) and NOW (Networks on Wheels) [4, 5, 6] have been formed to organize government, industrial and academic efforts to improve safety, enhance travel experience and bring information services to the traveler. WAVE (Wireless Access for Vehicular Environments) [7] is the IEEE 802.11p draft under development to define standards and protocols to enable communication between vehicles (V2V) and between vehicles and other infrastructure (V2I).

Delay tolerant networks (DTNs) [8], also known as Intermittently Connected Mobile Networks (ICMNs) or Opportunistic Networks, are characterized by periods of connectivity interspersed with periods where nodes are largely disconnected. Delay tolerant networking has found several applications in inter-planetary space communications, mobile ad hoc networks and sensor networks. Performance modeling in the context of ad hoc networks, particularly delay and throughput effects is of particular interest. An important observation is the absence of end-to-end connectivity in vehicular networks owing to the unique characteristics of vehicle mobility and time-varying vehicular density. While existing mobility models such as the Freeway and Manhattan model capture the mobility of vehicles along restricted pathways, they do not adequately reflect the fragmented connectivity. However, opportunistic connectivity allows us to employ a *store-carry-forward* mechanism, essentially a greedy approach.

In the context of vehicular networks, DTN messaging has been proposed in previous work in [3, 9, 10, 11]. The authors in references [12, 13] propose a model for evaluation of delay tolerant networking in vehicular networks. The model gives bounds on the performance of messaging in a vehicular network absent any infrastructure. The work demonstrates the gains achieved by delay tolerant messaging and the minimum density requirements. The model is essentially an infinite linear network and evaluates long-run average performance. In contrast, a network with access points is a finite case with unique messaging. In reference [10], the authors have evaluated vehicle traces on the highway and demonstrated that they closely follow exponential distribution of nodes. The work demonstrates network fragmentation and the impact of time varying vehicular traffic density on connectivity and hence, the performance of messaging.

The UMass DieselNET project explores the deployment of communication infrastructure over campus transportation network and records measurements on opportunistic networking [14]. Wu et al. have proposed an analytical model to represent a highway-vehicle scenario [9]. In their approach, they investigate speed differential between vehicles traveling in the same direction to bridge partitioned network of vehicles. An important distinction in our work is that we consider bidirectional connectivity which is intuitively faster due to the speed differential in traffic moving in opposing directions. In our work, we demonstrate that the transient connectivity offered by opposing traffic can provide a substantial improvement in message propagation rate.

The phase transition phenomenon in the context of ad hoc networks has been discussed in reference [15]. The authors discuss a model of random placement of nodes in a unit disk and analyse the probabilistic properties of the connectivity graph in the context of increasing communication radius. In reference [16], authors study the availability of transient paths of short hop-length in a mobile network and observe that a phase transition occurs as time and hops are jointly increased according to the logarithm of the network size. Several works have studied connectivity characteristics in a one-dimensional linear arrangement of nodes [17], [18], [19]. Our work is unique in that it considers a linear arrangement of nodes that are mobile in opposing directions. We model the transient connectivity and delay tolerance assumption that is unique from previous work. In reference [20], authors have demonstrated that mobility increases the capacity of an ad hoc wireless network. An analytical model developed by the authors demonstrates that for one-dimensional and random mobility patterns the interference decreases and often mobility aids in spreading the messages faster in the network. In a similar context, we demonstrate that under certain assumptions, increased mobility aids in message propagation as the network partitioning is bridged at a faster rate. However, the we note the interesting aspect where the increase is order of magnitude larger.

3 Vehicular Networking Environment

A network formed over moving vehicles has characteristics of topology and mobility that are unique from traditional mobile ad hoc networks. In this section, we describe the key observations and assumptions of the vehicular networking environment. We describe the highway environment, the nature of vehicle mobility and the time-varying density of vehicular traffic. We discuss the impact of these observations on the message exchange. Based on these observations, we describe a proposed messaging scheme that exploits the opportunistic connectivity between nodes to forward data. The messaging scheme forms the basis of the analytical model.

3.1 Highway Model

We consider a highway scenario where vehicles travel in either direction on a bi-directional roadway. We assume that vehicles are equipped with storage, computation and communication capabilities. The vehicles are modeled as point objects such that the length of a vehicle is not taken into account. Each vehicle is equipped with a GPS that enables location awareness and roadway information. Vehicles are able to exchange location information and other anonymous data such as speed and heading. The highway is modeled as linear while complex scenarios such as curvature in the roadway and intersections are resolved using GPS data. The roadway is annotated as *eastbound* and *westbound* for convenience in the narrative. The highway model is illustrated in Figure 1. A fixed radio range model is assumed such that vehicles are able to communicate with each other if the distance between them is less than radio range R. Vehicles tend to join and leave the highway at random. The expected behavior for a vehicle is to join a highway, travel for some period of time and then leave the roadway. We do not explicitly model the arrival and departure of vehicles from the highway, rather we model the density of vehicles on the roadway. As vehicles travel on the roadway, they come in intermittent contacts with vehicles traveling in opposite directions. We demonstrate that these opportunistic contacts can be utilized to aid message propagation.

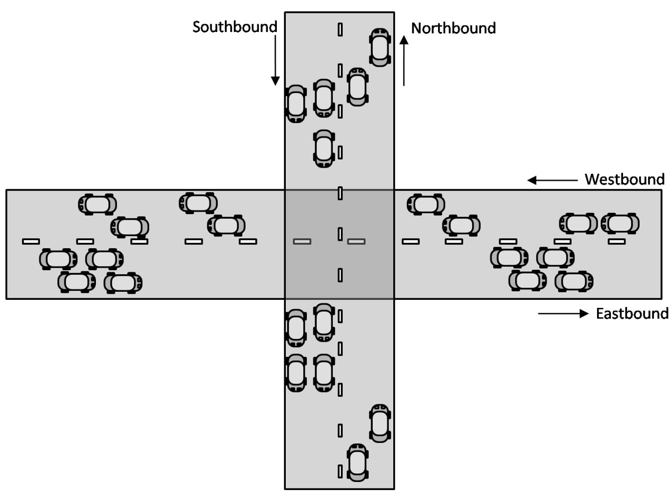


Figure 1: Illustration of the highway model.

3.2 Nature of Data Exchange

The nature of communication is a unique aspect of vehicular networking applications, different from other paradigms. Primarily, there are several types of data exchange in a VANET [1]. First, applications such as *safety messaging* are near-space applications where vehicles in close proximity, typically of the order of several hundred meters exchange status information to increase safety awareness. The goal for data exchange is to enable vehicles with enhanced safety systems to react to emergency conditions and avert accidents. The nature of the application requires strict latency constraints of the order of few milliseconds due to the time-critical nature of data and application. Second, *traffic* and *congestion monitoring* require collecting information from vehicles that span several kilometers. While the data are essential for trip planning, the latency requirements are relatively relaxed and the applications are delay tolerant. Finally, the *third* type of data is general purpose Internet access where vehicles are connected to a backbone network via road-side infrastructure such as access points. These three broad classifications are illustrated in Figure 2.

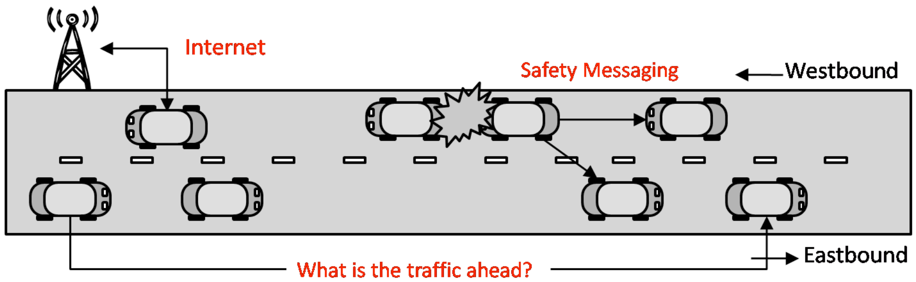


Figure 2: Three classifications of data exchange in vehicular networks.

An important observation in the vehicular networking space is the spatial temporal correlation of data. Data are generated as nodes traverse the network of roadways. An example is the traffic flow information on highways. Vehicles traveling on the highway generate statistics such as vehicle traffic flow, speed, throughput, etc. These data are related to a section of the highway that is traversed by the vehicle. At the same time these data are useful to vehicles that are approaching the section of the highway and are, at the time, some distance away. Thus, not only are the data spatially related, the source and destination of the data are spatially correlated on the roadway, separated by some distance. Furthermore, the data are temporally sensitive such that the lifetime of the data are limited and they become relevant after some time, as is in the case of traffic statistics that change with the time of the day. Thus, the vehicular network can be modeled as a spatial temporal database that continually generate data and at the same time consume the data. The nodes that form the source and destination of these data are potentially identified by their opportunistic location on the roadway.

3.3 Fragmentation of the Network

Vehicle traffic density on the roadway is a time-varying quantity. Mornings and evenings typically observe high traffic volumes on the roadway often leading to congestion ("rush hour"). At the other extreme, at nighttime, the roadways are usually deserted. Thus, the density of nodes in the network varies between the extremes of sparse and dense. Correspondingly, the network varies between connected and partitioned. Road traffic statistics and time-series snapshots of vehicular traffic have demonstrated that vehicles tend to travel in clusters on the roadway [21]. The clusters tend to be separated by some distance. Thus, in networking terms, the network is partitioned. The network is composed of disconnected sub-nets that are partitioned from each other. This is illustrated in Figure 3. However, the network topology changes at a constant rate as vehicles travel in opposing directions. Clusters come in intermittent contact with other clusters. Thus, sub-nets connect and disconnect frequently leading to time-varying partitioning. We term this time-varying partitioning as *fragmentation* of the network. In a network formed over moving vehicles, enabling messaging is challenging due to the absence of a fully connected network. The network is sparsely populated and there is lack of end-to-end connectivity in the network. MANET schemes that rely on end-to-end connectivity are a poor solution as a path from source to destination may not exist due to lack of sufficient node density in the network. If vehicle traffic traveling in opposing directions is included in path formation, the resulting paths are short-lived. Thus, routing schemes based on path formation strategies are an inefficient solution as a result of the increased overhead involved in path formation and path maintenance.

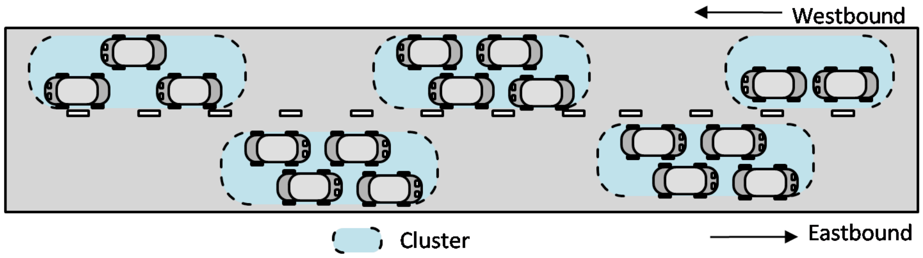


Figure 3: Illustration of clustering of vehicles on the roadway.

Thus, the requirement is of a messaging scheme that is able to adapt to the extremes of a sparse and dense node density and at the same time, solve the problem of time-varying network partitioning or *fragmentation*.

3.4 Messaging Model

In related work [3], we propose a messaging scheme that enables us to solve the problems of network *fragmentation*. A brief description of the scheme is provided here. The scheme relies on source and destination pairs identified on the basis of location. A common assumption in the VANET environment is GPS equipped vehicles that are location aware and share this information in a neighborhood. We propose to exploit the spatial-temporal correlation of data and nodes in the system. The data are identified as sourced from a location and destined for a location. The location coordinates obtained from GPS are embedded in each packet such that each packet is attributed (labelled). Thus, we are able to implement a simplified geographic routing protocol as each intermediate node forwards data based on its location and the source-destination locations embedded in the data packets. The scheme does not require the formation of an end-to-end path, rather each node is able to route based on the attributed data.

While the time-varying connectivity in the network presents a challenge to enable networking, it provides an opportunity to bridge the partitioning in the network. As vehicle traveling in one direction are likely to be partitioned, vehicles that are traveling in the opposing direction can be used as illustrated in Figure 4. This transient connectivity can be used irrespective of the direction of data transfer, *eastbound* or *westbound*.

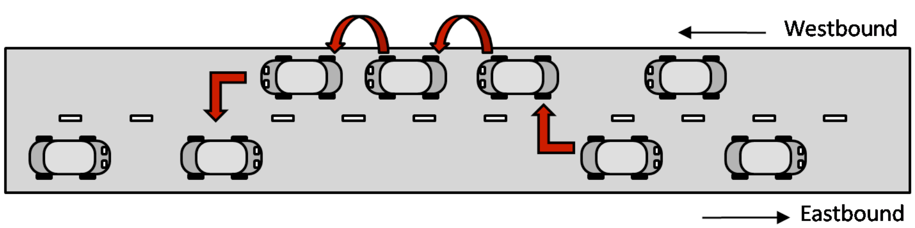
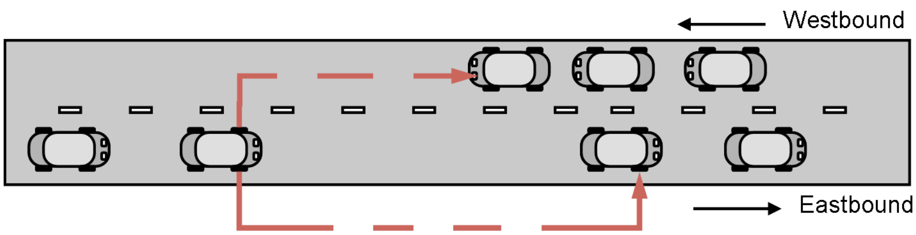


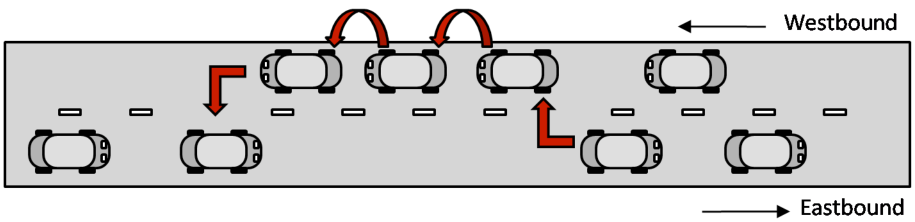
Figure 4: Partitions are bridged using vehicles traveling in the opposing direction.

However, it is important to note that this connectivity is not always instantaneously available. Partitions exist on either side of the roadway and in a sparse network there are large gaps between connected sub-nets. Here we propose the application of delay tolerant networking [8, **?**]. Delay Tolerant Networking (DTN) is essentially a *store-carry-forward* scheme where messages are cached or buffered in a node's memory when the network is disconnected. The data are forwarded as and when connectivity is available in the system. This is illustrated in Figure 5, where at the time of reference $t = 0$, the network is partitioned and there is lack of instantaneous connectivity between nodes. At time instant $t = \delta t$, the topology of the network changes by virtue of vehicle mobility and connectivity between previously partitioned nodes is available.

Thus, the proposed scheme is a connection-less messaging paradigm where data are exchanged as nodes come in intermittent contact.



(a) At $t = 0$, the network is partitioned and nodes are unable to communicate.



(b) At $t = \delta t$, topology changes, connectivity is achieved and vehicles are able to communicate.

Figure 5: Illustrating delay tolerant network (DTN) messaging as the network connectivity changes with time.

The application of delay tolerant networking and opportunistic connectivity create a unique messaging paradigm. There is transient connectivity in the network and messages are buffered in nodes in the absence of connectivity. The state of the network varies between the extremes of connected and disconnected. The messages are propagated multi-hop when the network is connected and buffered in the absence of connectivity. Thus, the performance of the messaging protocol also varies as the connectivity. We develop an analytical model that captures the essence of time varying connectivity and the corresponding performance of the messaging scheme.

4 Analysis

The vehicular networking environment, as described in Section 3, is one that has time varying connectivity and demonstrates *fragmentation*. We proposed to exploit the intermittent connectivity offered by traffic in opposing directions to bridge the partitioning in the network. An adaptive messaging scheme applies the concepts of delay tolerant networking where messages are propagated multi-hop when the network is connected and are buffered when there is lack of connectivity. The performance of the messaging scheme alternates as the network experiences time-varying connectivity. We develop an analytical model that considers a simplified VANET environment as a one-dimensional linear model. Subsequent sections present the simplified model and the associated notation to ease the explanation. The model captures the partitioning in the network with vehicular traffic density, vehicle and the physical radio as parameters. While an exact analysis of the connectivity is hard to develop as a result of the vehicular mobility in opposing directions, we describe a discretization of the model that allows us to capture the connectivity model. We derive an *upper* bound and a *lower* bound for the messaging performance based on the discretization. Finally, we develop an *approximation* that closely follows the simulation results and can be used to evaluate and study the system parameters of vehicle traffic density, transmission range, etc.

4.1 Model and Notation

We consider a bi-directional roadway scenario wherein vehicles, also referred to as nodes, travel in either *eastbound* or *westbound* directions, as illustrated in Figure 6. Vehicles are assumed to be point objects such that the length of a vehicles is not taken into account while computing distance. The model is a linear one-dimensional approximation of the roadway absent any infrastructure, such that vehicles form nodes of a linear ad hoc network. In each direction, nodes are assumed to move at a constant speed v m/s such that the distance between nodes moving along the same direction remains unchanged. We assume a fixed transmission range R. Thus, two nodes are directly connected by a radio link if the distance between them is R or less. The distance X between any two consecutive nodes is an i.i.d. exponential random variable, with parameter λ_e for *eastbound* traffic and λ_w for *westbound* traffic. The exponential distribution has been shown to be in good agreement with real vehicular traces under uncongested traffic conditions, e.g., fewer than 1000 vehicles per hour [**?**]. Our work focuses on that particular scenario, where as vehicular traffic moves in opposite directions, periods of connectivity alternate with periods of disconnection.

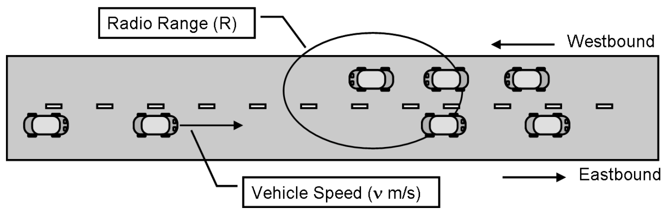


Figure 6: Illustration of the highway model.

To evaluate the performance of the messaging scheme, we consider the metric *effective message propagation speed*, denoted by v_{eff}. We evaluate the performance of messaging as the physical distance covered in unit time or the *message propagation speed* similar to vehicle speed. As data and nodes are spatially correlated, we aim to evaluate the time taken for a message to propagate given physical distance, and hence, the speed. With the assumption of delay tolerance in the network, data messages are buffered at nodes until connectivity becomes available. There are alternating periods of disconnection and (multi-hop) connectivity. We refer to the alternating periods of disconnection and (multi-hop) connectivity as *phase 1* and *phase 2*, respectively. In phase 1, when nodes are disconnected, data propagate at vehicle speed v, waiting for connectivity to be renewed. In phase 2, when multi-hop connectivity is available, data propagate at radio speed v_{radio}. This speed is determined by the characteristics of the physical and network layers. The multi-hop radio propagation speed is order of magnitude larger than the vehicle speed, i.e. $v_{radio} >> v$. Thus, the *effective message propagation speed* (v_{eff}) is a function of the time spent in the two alternating phases.

Given the characteristics of the physical radio, we evaluate *radio speed* (v_{radio}) as the rate at which a message is propagated by the radio. For radio range R, and considering propagation and transmission delays as τ, $v_{radio} = R/\tau$. As data messages are buffered when nodes are disconnected, they cover a physical distance as the vehicle travels, at a rate equivalent to vehicle speed v m/s. Thus, the *effective propagation speed* is a function of the distance covered at *radio speed* (v_{radio} m/s) and at vehicle speed (v m/s). A typical value is $v_{radio} = 1000$ m/s, as obtained from measurements [].

This system can be modeled as an *alternating renewal process* [22], where message propagation cyclically alternates between phases 1 and 2. Denote by T_1^n and T_2^n the (random) amounts of time a message spends in these two phases, during the n-th cycle. The random vectors $(T_1^n, T_2^n), n \geq 1$ are i.i.d., due to the memoryless assumption on the inter-vehicular distances. Note, however, that T_1^n and T_2^n are not independent. Denoting $E[T_1] = E[T_1^n]$ the expected time spent in phase 1 and $E[T_2] = E[T_2^n]$ the expected time spent in phase 2, we obtain from Theorem 3.4.4 in [22] that the long-run fraction of time spent in each of these states is respectively

$$p_1 = \frac{E[T_1]}{E[T_1] + E[T_2]}; \qquad p_2 = \frac{E[T_2]}{E[T_1] + E[T_2]}. \tag{1}$$

From Eq. (1), it follows that

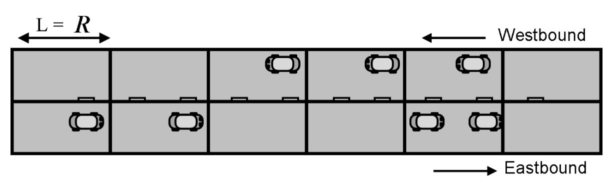
$$\begin{aligned} v_{eff} &= p_1 v + p_2 v_{radio} && (2) \\ &= \frac{E[T_1]v + E[T_2]v_{radio}}{E[T_1] + E[T_2]}. && (3) \end{aligned}$$

The primary goal of our analysis is to determine how $E[T_1]$ and $E[T_2]$ (and thereby the effective message propagation speed v_{eff}) depend on the parameters λ_e, λ_w, R, v, and v_{radio}. Since the derivation of exact expressions appears involved, we instead focus next our efforts on the derivation of upper and lower bounds on these quantities.

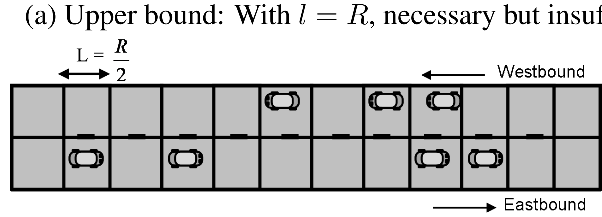
4.2 Discretization

The analysis of the problem at hand is rendered difficult by its continuous nature. To circumvent this difficulty, we provide bounds by discretizing each side of the roadway into cells, each of size l. We consider a cell to be *occupied* if a vehicle is positioned within that cell. By virtue of the exponential distribution of vehicle traffic and exploiting the *memoryless property*, we are able to compute the probability (p) that a node is *occupied* as: $p = (1 - e^{-\lambda l})$, where l is the cell size and λ is the density of traffic distribution. Within each cell, the vehicle or node maybe located in the interval $(0, l)$.

We consider two values for the cell size: $R/2$ and R, as shown in Fig. 7. When the cell size is $R/2$, the nodes in adjacent cells are surely connected irrespective of their location within the respective cells. Even when the nodes are located at the two extremes of adjacent cells, the maximum distance between the nodes is R, which is within the communication range. Thus, if we require each adjacent cell of length $R/2$ to be occupied by at least one node to ensure connectivity, then one can obtain a lower bound on v_{eff} as detailed below.



(a) Upper bound: With $l = R$, necessary but insufficient condition.



(b) Lower bound: With $l = R/2$, sufficient but not always necessary condition.

Figure 7: Illustrating the discretization of node distribution on the roadway, *upper* and *lower* bounds for connectivity.

Conversely, if we require each adjacent cell of length R to be occupied by at least one node as a condition for connectivity, then one can derive an upper bound on v_{eff}. Here, the distance between two nodes in adjacent cells may vary within the range 0 and $2R$. Thus, the nodes may or may not be connected. We note that the lower bound requirement is a sufficient but not always necessary condition for connectivity since two nodes may be connected even if an empty cell separates between them. On the other hand, with cells of length R, nodes in adjacent cells are not guaranteed to be always connected, e.g, if the nodes are located at the opposite ends of each cell. Hence, the upper bound is a necessary but insufficient condition. The two cases are illustrated in Figure 7.

4.3 Connectivity

The traffic along *eastbound* and *westbound* direction is distributed exponentially with parameters λ_e and λ_w respectively. The inter-node distance between adjacent *eastbound* nodes is given by: $f(x) = \lambda_e e^{-\lambda_e x}$. The probability that consecutive (adjacent) nodes along *eastbound* are connected is given by: $P(X_i < R) = (1 - e^{-\lambda_e R})$. Similarly, *westbound* nodes are connected with probability given by: $P(X_i < R) = (1 - e^{-\lambda_w R})$. If the distance between the two nodes *eastbound* is greater than R, i.e. $(X_i > R)$, connectivity must be achieved using nodes along *westbound* direction. As per the discretization described in Section 4.2, the distance X_i is equivalent to, say, n cells. The nodes along *eastbound* are connected if the each of the n cells is occupied by at least one node, an event which occurs with probability $(1 - e^{-\lambda_w R})^n$.

In the event that each of the n cells in the *westbound* is not occupied by a node, the nodes along *eastbound* are deemed to be disconnected. The data are buffered in a node's cache until the connectivity is achieved again, which is the phase 1 of message propagation. The node and hence, the data traverse some distance (cells) until connectivity is achieved. The number of cells traversed until connectivity is achieved is analogous to the number of trials until a sequence is seen. This is described as *pattern matching* in classical probability theory [22]. The pattern matching problem describes the task to compute the expected number of trials until n consecutive successes. It is analogous to our problem as we try to find the number of cells traversed by a node until n consecutive cells along *westbound* traffic are occupied by one or more nodes. From known results on *pattern matching* [22], the expected number of trials until n consecutive successes is given is:

$$E[N] = \frac{1 - p^n}{(1 - p)p^n} \tag{4}$$

where p is the probability of success for a single event. This result gives us the expected number of trials until after the *pattern matching* is achieved. Hence, we modify this result suitably to compute the distance until before the pattern is observed to compute the distance (cells) until connectivity is achieved and hence, the distance traversed in phase 1.

Thus, for the discrete model, we are able to derive the probability for connectivity and hence, the expectation of distance until connectivity is achieved in delay tolerant network setting. These enable us to evaluate the message propagation rate as the network alternates between connected and disconnected states. Further, we will derive the *upper* and *lower* bound for the message propagation rates.

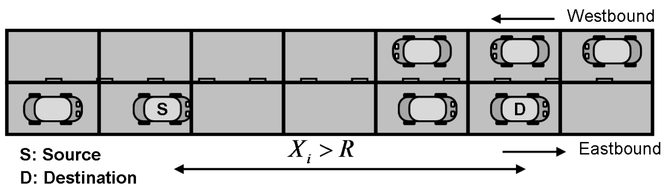
4.4 Upper Bound

We first consider the case of cells of size R. We refer to this system as *System* u. We denote by $E[T_1]_u$ and $E[T_2]_u$, the expected time spent by *System* u in phases 1 and 2, respectively. Similarly, $E[D_1]_u = vE[T_1]_u$ and $E[D_2]_u = v_{radio}E[T_2]_u$ represent the expected distance. In the following, we derive a lower bound on $\underline{E[T_1]_u}$ and an upper bound on $\overline{E[T_2]_u}$, which by Eq. (3) leads to an upper bound on v_{eff}.

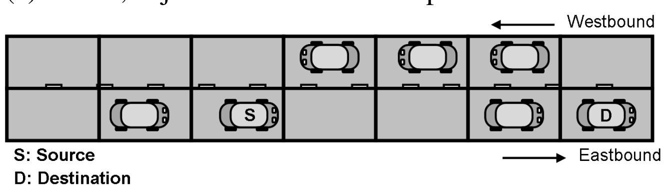
Phase 1

In phase 1, there is absence of multi-hop connectivity along *eastbound* or *westbound* traffic, as illustrated in Figure 8(a). The *source* and the *destination* are disconnected as they separated by distance $X_i > R$.

Correspondingly, the adjacent cells between the source and the destination are unoccupied at the given instant. As nodes move, there is opportunistic connectivity over nodes along *westbound* traffic. The nodes *eastbound* are connected when each of the cells *westbound* in the gap is occupied by one or more nodes (Figure 8(a)). The gap between *eastbound* nodes (x) is divided in cells, each of size R. The number of cells in the gap x denoted k is lower bounded by $\lfloor x/R \rfloor$. We must chose a lower bound, so that the expected time elapsing until connectivity is lower bounded. The rate of data propagation in this phase is low, thus, to find the upper bound for the effective propagation rate, we need to lower bound the time spent in this phase.



(a) At t=0, adjacent cells are unoccupied and the network is disconnected.



(b) at t=τ, adjacent cells are occupied as nodes move and there is connectivity over *westbound* traffic.

Figure 8: Illustrating the *phase 1* and *phase 2* of message propagation.

The lower bound probability that nodes are disconnected ($\bar{C}$) for given inter-node separation (x), cell size (R), and lower bound for number of cells ($k = \lfloor x/R \rfloor$) is evaluated as:

$$\underline{P(\bar{C}|X = x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - (1 - e^{-\lambda_w R})^{\lfloor x/R \rfloor} & \text{if } x > R \end{cases} \tag{5}$$

Substituting $p = (1 - e^{-\lambda_w R})$:

$$\underline{P(\bar{C}|X = x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - p^{\lfloor x/R \rfloor} & \text{if } x > R \end{cases} \tag{6}$$

The lower bound probability that nodes are disconnected ($\bar{C}$, independent of node distribution, is evaluated as:

$$
\begin{aligned}
\underline{P(\bar{C})} &= \int_0^\infty P(\bar{C}|X=x) f_X(x) dx && (7)\\
&= \int_R^\infty (1 - p^{\lfloor x/R \rfloor}) \lambda_e e^{-\lambda_e x} dx && (8)\\
&= \sum_{n=1}^{\infty} (1-p^n) \int_{nR}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx && (9)\\
&= \sum_{n=1}^{\infty} (1-p^n)(e^{-\lambda_e R(n)} - e^{-\lambda_e R(n+1)}) && (10)\\
&= (1-e^{-\lambda_e R}) \sum_{n=1}^{\infty} (1-p^n)(e^{-\lambda_e R(n)}) && (11)\\
&= (1-e^{-\lambda_e R}) \left[\sum_{n=1}^{\infty} (e^{-\lambda_e R(n)}) - \sum_{n=1}^{\infty} p^n)(e^{-\lambda_e R(n)}) \right] && (12)\\
&= (1-e^{-\lambda_e R}) \left(\frac{e^{-\lambda_e R}}{1-e^{-\lambda_e R}} - \frac{p e^{-\lambda_e R}}{1 - p e^{-\lambda_e R}} \right) && (13)
\end{aligned}
$$

Equation (8) is derived by substituting from equation (6). Equation (122) is derived by adjusting the limits of integral in equation (8). By integrating and expressing as sum of an infinite series, we obtain (10). Finally, the equation (13) is obtained by evaluating the sum of infinite series. This expression gives us the lower bound probability of nodes being disconnected in the *System u*.
The result in Equation (13) is the sum of an infinite series which converges when $e^{-\lambda_e R}(1 - e^{-\lambda_w R}) < 1$. The corresponding lower bound for the density function is evaluated as:

$$
\begin{aligned}
\underline{f_{X|\bar{C}}(x)} &= \frac{f_X(x) \underline{\underline{P(\bar{C}|X=x)}}}{P(\bar{C})}\\
&= \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - (1 - e^{-\lambda_w R})^{\lfloor x/R \rfloor}) \right) & \text{if } x > R \end{cases}
\end{aligned}
$$

Substituting $p = (1 - e^{-\lambda_w R})$, we get:

$$
\underline{f_{X|\bar{C}}(x)} = \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - p^{\lfloor x/R \rfloor}) \right) & \text{if } x > R \end{cases} \qquad (14)
$$

The expected number of cells traversed until connectivity is achieved is analogous to the *pattern matching* problem described in Section 4.3. Thus, applying Equation (4) to find the lower bound for expected distance for a given separation $X_i = x$, cell size R, we obtain the following expression:

$$
\underline{E[D_1|X=x]} = \left[\frac{1 - p^{\lfloor x/R \rfloor}}{(1-p)p^{\lfloor x/R \rfloor}} - \left\lfloor \frac{x}{R} \right\rfloor \right] \frac{R}{2} \qquad (15)
$$

The above expression in Equation (15) signifies the distance traversed until connectivity *westbound* traffic is achieved. This distance is computed by finding the lower bound for expected number of cells until connectivity and finding the corresponding distance, given the cell size is R. Note that a correction factor of $1/2$ is applied as nodes in either direction *westbound* and *eastbound* are traveling at speed vm/s. Thus, the required distance is effectively halved. Furthermore, the expression in Equation (4) gives us the cells until after the pattern is seen. The number of cells traversed until before the pattern is seen is the expected number of cells minus the desired number of cells. Hence, we subtract $\lfloor x/R \rfloor$ to find the distance until connectivity.

Generalizing the result in Equation (15) for the exponential distribution of nodes:

$$
\begin{aligned}
\underline{E[D_1]_u} &= \int_0^\infty \underline{E[D_1|X] f_{X|\bar{C}}(x)} dx && (16)\\
&= \int_R^\infty \frac{E[D_1|X] f_X(x) P(\bar{C}|X=x)}{P(\bar{C})} dx && (17)\\
&= \frac{1}{P(\bar{C})} \int_R^\infty \left[\left(\frac{1-p^{\lfloor x/R \rfloor}}{(1-p)p^{\lfloor x/R \rfloor}} - \left\lfloor \frac{x}{R} \right\rfloor \right) \frac{R}{2} \right] \lambda_e e^{-\lambda_e x} (1-p^{\lfloor x/R \rfloor}) dx && (18)\\
&= \frac{R}{2P(\bar{C})} \left[\int_R^\infty \frac{(1-p^{\lfloor x/R \rfloor})^2}{(1-p)p^{\lfloor x/R \rfloor}} \lambda_e e^{-\lambda_e x} dx - \int_R^\infty \left\lfloor \frac{x}{R} \right\rfloor \lambda_e e^{-\lambda_e x} (1-p^{\lfloor x/R \rfloor}) dx \right] && (19)\\
&= \frac{R}{2P(\bar{C})} \left[\frac{1}{1-p} \sum_{n=1}^\infty \frac{(1-p^n)^2}{p^n} \int_{nR}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx - \sum_{n=1}^\infty n(1-p^n) \int_{nR}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx \right] && (20)\\
&= \frac{R}{2P(\bar{C})} \left[\frac{1}{1-p} \sum_{n=1}^\infty \frac{(1-p^n)^2}{p^n} (e^{-\lambda_e nR} - e^{-\lambda_e (n+1)R}) - \sum_{n=1}^\infty n(1-p^n)(e^{-\lambda_e nR} - e^{-\lambda_e (n+1)R}) \right] && (21)\\
&= \frac{R(1-e^{\lambda_e R})}{2P(\bar{C})} \left[\frac{1}{1-p} \sum_{n=1}^\infty \frac{(1-p^n)^2}{p^n} e^{-\lambda_e nR} - \sum_{n=1}^\infty n(1-p^n) e^{-\lambda_e nR} \right] && (22)\\
&= \frac{R(1-e^{\lambda_e R})}{2P(\bar{C})} \left[\frac{1}{1-p} \left[\frac{e^{-\lambda_e R}}{p-e^{-\lambda_e R}} + \frac{pe^{-\lambda_e R}}{1-pe^{-\lambda_e R}} - \frac{2e^{-\lambda_e R}}{1-e^{-\lambda_e R}} \right] - \left[\frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})^2} - \frac{pe^{-\lambda_e R}}{(1-pe^{-\lambda_e R})^2} \right] \right] && (23)
\end{aligned}
$$

Equation (17) is derived by substituting from equation (14). Equation (20) is derived by adjusting the limits of integral in equation (19). By integrating and expressing as sum of an infinite series, we obtain (21). Finally, the equation (23) is obtained by evaluating the sum of infinite series. This expression gives us the average distance of data propagation in Phase 1.
The result in Equation (23) is the sum of an infinite series which converges when $e^{-\lambda_e R} < (1 - e^{-\lambda_w R})$ and $e^{-\lambda_e R}(1 - e^{-\lambda_w R}) < 1$.

Phase 2

Once connectivity is achieved, the gap between *eastbound* nodes, ($X_i > R$), is *bridged* by *westbound* nodes. The distance is covered at at multi-hop radio speed v_{radio} m/s. We compute the upper-bound for the expected distance between *eastbound* nodes given that the nodes were disconnected. This distance is covered after Phase 1 once connectivity is achieved. To derive an upper bound limit, we compute the upper bound probability that nodes are disconnected.

$$\overline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - (1-e^{-\lambda_w R})^{\lceil x/R \rceil} & \text{if } x > R \end{cases} \tag{24}$$

Substituting $p = (1 - e^{-\lambda_w R})$:

$$\overline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - p^{\lceil x/R \rceil} & \text{if } x > R \end{cases} \tag{25}$$

The upper bound probability that nodes are disconnected ($\bar{C}$), independent of node distribution, is evaluated as:

$$\begin{aligned}
\overline{P(\bar{C})} &= \int_0^\infty P(\bar{C}|X=x) f_X(x)dx && (26)\\
&= \int_R^\infty (1-p^{\lceil x/R \rceil})\lambda_e e^{-\lambda_e x} dx && (27)\\
&= \sum_{n=1}^{\infty}(1-p^{n+1})\int_{nR}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx && (28)\\
&= \sum_{n=1}^{\infty}(1-p^{n+1})(e^{-\lambda_e R(n)} - e^{-\lambda_e R(n+1)}) && (29)\\
&= (1-e^{-\lambda_e R})\sum_{n=1}^{\infty}(1-p^{n+1})(e^{-\lambda_e R(n)}) && (30)\\
&= (1-e^{-\lambda_e R})\left[\sum_{n=1}^{\infty}(e^{-\lambda_e R(n)}) - p\sum_{n=1}^{\infty} p^n)(e^{-\lambda_e R(n)})\right] && (31)\\
&= (1-e^{-\lambda_e R})\left(\frac{e^{-\lambda_e R}}{1-e^{-\lambda_e R}} - \frac{p^2 e^{-\lambda_e R}}{1-pe^{-\lambda_e R}}\right) && (32)
\end{aligned}$$

Equation (27) is derived by substituting from equation (25). Equation (28) is derived by adjusting the limits of integral in equation (27). By integrating and expressing as sum of an infinite series, we obtain (29). Finally, the equation (32) is obtained by evaluating the sum of infinite series. This expression gives us the upper bound probability of nodes being disconnected in the *System u*.
The result in Equation (32) is the sum of an infinite series which converges when $e^{-\lambda_e R} < (1-e^{-\lambda_w R})$ and $e^{-\lambda_e R}(1-e^{-\lambda_w R}) < 1$.
The corresponding upper bound for the density function can be expressed as:

$$\begin{aligned}
\overline{f_{X|\bar{C}}(x)} &= \frac{f_X(x)\overline{P(\bar{C}|X=x)}}{P(\bar{C})}\\
&= \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})}\left(\lambda_e e^{-\lambda_e x}(1-(1-e^{-\lambda_w R})^{\lceil x/R \rceil})\right) & \text{if } x > R \end{cases}
\end{aligned}$$

Substituting $p = (1-e^{-\lambda_w R})$, we get:

$$\overline{f_{X|\bar{C}}(x)} = \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})}\left(\lambda_e e^{-\lambda_e x}(1-p^{\lceil x/R \rceil})\right) & \text{if } x > R \end{cases} \tag{33}$$

Given the density function (Eq. (33)), the upper bound for average separation between *eastbound* nodes, given that they are not connected is evaluated as:

$$\overline{E[X|\bar{C}]}=\int_R^\infty x\overline{f_{X|\bar{C}}(x)}dx \tag{34}$$

$$=\int_R^\infty \frac{xf_X(x)\overline{P(\bar{C}|X=x)}}{\overline{P(\bar{C})}}dx \tag{35}$$

$$=\int_R^\infty \frac{x\lambda_e e^{-\lambda_e x}(1-p^{\lceil x/R \rceil})}{P(\bar{C})}dx \tag{36}$$

$$=\frac{1}{P(\bar{C})}\sum_{n=1}^{\infty}(1-p^{n+1})\int_{nR}^{(n+1)R} x\lambda_e e^{-\lambda_e x}dx \tag{37}$$

$$=\frac{1}{\lambda_e P(\bar{C})}\sum_{n=1}^{\infty}(1-p^{n+1})\left[(1+\lambda_e nR)e^{-\lambda_e nR}-(1+\lambda_e(n+1)R)e^{-\lambda_e(n+1)R}\right] \tag{38}$$

$$=\frac{1}{\lambda_e P(\bar{C})}\sum_{n=1}^{\infty}(1-p^{n+1})(e^{-\lambda_e Rn})\left[1+\lambda_e R(n)-(1+\lambda_e R(n+1))e^{-\lambda_e R}\right] \tag{39}$$

$$=\frac{1}{\lambda_e P(\bar{C})}\sum_{n=1}^{\infty}(e^{-\lambda_e R})^n\left[(1-e^{-\lambda_e R})(1+\lambda_e R(n))-\lambda_e R(e^{-\lambda_e R})\right]$$

$$-\sum_{n=1}^{\infty}p^{n+1}(e^{-\lambda_e R})^n\left[(1-e^{-\lambda_e R})(1+\lambda_e R(n))-\lambda_e R(e^{-\lambda_e R})\right] \tag{40}$$

$$=\frac{1}{\lambda_e P(\bar{C})}\left[e^{-\lambda_e R}(1+\lambda_e R)-\frac{p^2(e^{-\lambda_e R})(1-e^{-\lambda_e R})}{1-pe^{-\lambda_e R}}-\frac{p^2\lambda_e R(e^{-\lambda_e R})(1-e^{-\lambda_e R})}{(1-pe^{-\lambda_e R})^2}+\frac{p^2\lambda_e R(e^{-\lambda_e R})^2}{(1-pe^{-\lambda_e R})}\right] \tag{41}$$

Equation (35) is derived by substituting from equation (33). Equation (36) is derived by adjusting the limits of integral in equation (35). By integrating and expressing as sum of an infinite series, we obtain (38). Finally, the equation (41) is obtained by evaluating the sum of infinite series. This expression gives us the upper bound probability of nodes being disconnected in the *System u*.
The result in Equation (41) is the sum of an infinite series which converges when $e^{-\lambda_e R} < (1-e^{-\lambda_w R})$ and $e^{-\lambda_e R}(1-e^{-\lambda_w R}) < 1$.

In phase 2, the nodes are connected by multi-hop connectivity and the data are able to propagate multi-hop over connected nodes. As illustrated in Figure 9, the messages travel along *eastbound* nodes when connectivity along *eastbound* nodes is available. When the nodes *eastbound* are disconnected the messages are forwarded by nodes traveling in the *westbound* direction to the next hop *eastbound*.

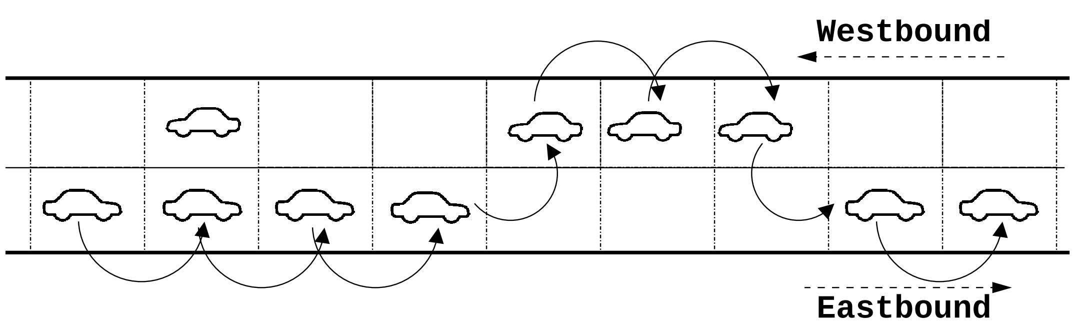


Figure 9: Illustration of connectivity in Phase 2 of data propagation.

The upper bound probability of connectivity (C) for nodes for given value of inter-node distance ($X_i = x$)

can be expressed as:

$$\overline{P(C|X=x)} = \begin{cases} 1 & \text{if } x \leq R, \\ (1-e^{-\lambda_w R})^{\lfloor x/R \rfloor} & \text{if } x > R \end{cases} \tag{42}$$

The nodes are connected if the distance between nodes is within the communication range ($x \leq R$). In the event that the distance is beyond the communication range ($x > R$), connectivity over *westbound* traffic is sought. The distance corresponds to at least $\lfloor x/R \rfloor$. The nodes along *eastbound* are connected if each of these cells is occupied by a node. The upper bound probability of this event is $((1-e^{-\lambda_w R})^{\lfloor x/R \rfloor})$ Thus, the probability of connectivity for nodes along *eastbound* traffic $P(C)$, independent of inter-node distance X, is derived as:

$$\begin{aligned}
\overline{P(C)} &= \int_0^\infty \overline{P(C|X=x)} f_X(x)dx && (43) \\
&= \int_0^R 1.\lambda_e e^{-\lambda_e x}dx + \int_R^\infty (1-e^{-\lambda_w R})^{\lfloor x/R \rfloor} \lambda_e e^{-\lambda_e x}dx && (44) \\
&= \int_0^R \lambda_e e^{-\lambda_e x}dx + \int_R^{2R} (1-e^{-\lambda_w R})\lambda_e e^{-\lambda_e x}dx + \int_{2R}^{3R} (1-e^{-\lambda_w R})^2 \lambda_e e^{-\lambda_e x}dx + \ldots \\
&+ \ldots + \int_{(n-1)R}^{nR} (1-e^{-\lambda_w R})^{n-1} \lambda_e e^{-\lambda_e x}dx + \ldots && (45) \\
&= \sum_{n=0}^{\infty} (1-e^{-\lambda_w R})^n (e^{-\lambda_e R})^n (1-e^{-\lambda_e R}) && (46) \\
&= \frac{(1-e^{-\lambda_e R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})} && (47)
\end{aligned}$$

Equation (44) is derived by substituting from equation (42). Equation (45) is derived by adjusting the limits of integral in equation (44). By integrating and expressing as sum of an infinite series, we obtain Equation (46). Finally, the equation (47) is obtained by evaluating the sum of infinite series.
The result in Equation (32) is the sum of an infinite series which converges when $e^{-\lambda_e R}(1-e^{-\lambda_w R}) < 1$.

Given the connectivity, we can derive the physical distance of data propagation for a given distribution. The data propagation is conditional upon the density function of node distribution. Thus, the propagation distance is a function of connectivity along *eastbound* and *westbound* node and the density function. The upper bound for density function for data propagation distance is expressed as:

$$\begin{aligned}
\overline{f_{X|C}(x)} &= \frac{\overline{f_X(x)P(C|X=x)}}{\overline{P(C)}} \\
&= \begin{cases} \frac{1}{P(C)}\left(\lambda_e e^{-\lambda_e x}\right) & \text{if } x \leq R \\ \frac{1}{P(C)}\left(\lambda_e e^{-\lambda_e x}(1-e^{-\lambda_w R})^{\lfloor x/R \rfloor}\right) & \text{if } x > R \end{cases} && (48)
\end{aligned}$$

Thus, the upper bound for the expected distance of data propagation in Phase 2 is evaluated as:

$$\overline{E[X|C]} = \int_0^\infty x\overline{f_{X|C}(x)}dx \tag{49}$$

$$= \int_0^\infty \frac{x f_X(x) P(C|X=x) dx}{P(C)} \tag{50}$$

$$= \frac{1}{P(C)} \left[\int_0^R \lambda_e x e^{-\lambda_e x} dx + \int_R^\infty \lambda_e x e^{-\lambda_e x} (1 - e^{-\lambda_w R})^{\lfloor x/R \rfloor} dx \right] \tag{51}$$

$$= \frac{1}{P(C)} \left[\int_0^R \lambda_e e^{-\lambda_e x} x dx + \int_R^{2R} (1 - e^{-\lambda_w R}) \lambda_e e^{-\lambda_e x} x dx + \int_{2R}^{3R} (1 - e^{-\lambda_w R})^2 \lambda_e e^{-\lambda_e x} x dx \right. \tag{52}$$

$$\left. + \ldots + \int_{(n-1)R}^{nR} (1 - e^{-\lambda_w R})^{n-1} \lambda_e e^{-\lambda_e x} x dx + \ldots \right]$$

$$= \sum_{n=0}^{\infty} \frac{1}{P(C)} \frac{1}{\lambda_e} \left[(1 - e^{\lambda_w R})^n (e^{-\lambda_e R})^n \left(1 + \lambda_e n R - (1 + \lambda_e (n+1) R) e^{-\lambda_e R} \right) \right] \tag{53}$$

$$= \frac{1}{P(C)} \frac{1}{\lambda_e} \left[\frac{1 - (1 + \lambda_e R) e^{-\lambda_e R}}{1 - e^{-\lambda_e R}(1 - e^{-\lambda_w R})} + \frac{\lambda_e R e^{-\lambda_e R} (1 - e^{-\lambda_w R})(1 - e^{-\lambda_e R})}{(1 - e^{-\lambda_e R}(1 - e^{-\lambda_w R}))^2} \right] \tag{54}$$

Equation (50) is derived by substituting from equation (48). Equation (52) is derived by adjusting the limits of integral in equation (51). By integrating and expressing as sum of an infinite series, we obtain Equation (53). Finally, the equation (54) is obtained by evaluating the sum of infinite series. This expression gives us an upper bound for the distance of message propagation when there is multi-hop connectivity is available in Phase 2.
The result in Equation (32) is the sum of an infinite series which converges when $e^{-\lambda_e R}(1 - e^{-\lambda_w R}) < 1$.

$$\overline{E[D_2]_u} = \frac{\overline{E[X|C]P(C)}}{1 - \overline{P(C)}} \tag{55}$$

$$= \frac{1}{\lambda_e} \left(\frac{1 - e^{-\lambda_e R}(1 - e^{-\lambda_w R})}{e^{-\lambda_e R} e^{-\lambda_w R}} \right) \left[\frac{1 - (1 + \lambda_e R) e^{-\lambda_e R}}{1 - e^{-\lambda_e R}(1 - e^{-\lambda_w R})} + \frac{\lambda_e R e^{-\lambda_e R} (1 - e^{-\lambda_w R})(1 - e^{-\lambda_e R})}{(1 - e^{-\lambda_e R}(1 - e^{-\lambda_w R}))^2} \right] \tag{56}$$

4.5 Lower Bound

We now consider the lower bound limit for cell size $R/2$. We refer to this system as *System v*. We denote by $E[T_1]_v$ and $E[T_2]_v$, the expected time spent by system *v* in phases 1 and 2, respectively. Similarly, $E[D_1]_v = vE[T_1]_v$ and $\underline{E[D_2]_v} = v_{radio} E[T_2]_v$ represent the expected distance. In the following, we derive an upper bound on $\overline{E[T_1]_v}$ and a lower bound on $\underline{E[T_2]_v}$, which by Eq. (3) leads to a lower bound on v_eff.

As explained previously, the lower bound for the cell size is a sufficient condition for connectivity but not always necessary. As a consequence of the cell size, we need to recompute the corresponding density functions and hence, the expected value data propagation rates.

Phase1

In phase 1, there is absence of multi-hop connectivity along either the *eastbound* or *westbound* traffic, data are cached in nodes as they travel at vehicle speed vm/s. The distance between *eastbound* nodes is greater than R, there is opportunistic connectivity over *westbound* traffic. For the lower bound, the gap x along *westbound* is divided in cells of size $R/2$. This ensures that once each cell along *westbound* is occupied by a node, the *eastbound* nodes are surely connected. This describes the sufficient but not necessary condition as there can be more nodes than required for connectivity. Thus, the number of cells (k) in the gap (x) is given by: $k = |2x/R|$.

To compute an upper bound on $E[T_1]_v$, in contrast to the derivation in 4.4, we compute an upper bound on the probability that nodes *eastbound* are disconnected ($\bar{C}$), to maximise the distance covered in Phase 1 ($E[D_1]_v$). The upper bound probability that nodes are disconnected ($\bar{C}$) for given inter-node distance (x), cell size ($R/2$), and upper bound or number of cells ($k = \lceil 2x/R \rceil$) is given by:

$$\overline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - (1 - e^{-\lambda_w R/2})^{\lceil 2x/R \rceil} & \text{if } x > R \end{cases} \tag{57}$$

Substituting $p_v = (1 - e^{-\lambda_w R/2})$:

$$\overline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - p_v^{\lceil 2x/R \rceil} & \text{if } x > R \end{cases} \tag{58}$$

The upper bound probability that nodes are disconnected ($\bar{C}$), independent of node distribution, is evaluated as:

$$\begin{aligned}
\overline{P(\bar{C})} &= \int_R^\infty \overline{P(\bar{C}|X=x)} f_X(x) dx && (59)\\
&= \int_R^\infty (1 - p_v^{\lceil 2x/R \rceil}) \lambda_e e^{-\lambda_e x} dx && (60)\\
&= \sum_{n=1}^{\infty} (1 - p_v^{2n+1}) \int_{nR}^{(2n+1)R/2} \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} (1 - p_v^{2n+2}) \int_{(2n+1)R/2}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx && (61)\\
&= \sum_{n=1}^{\infty} (1 - p_v^{2n+1})(e^{-\lambda_e R(n)} - e^{-\lambda_e R/2(2n+1)}) + \sum_{n=1}^{\infty} (1 - p_v^{2n+2})(e^{-\lambda_e R/2(2n+1)} - e^{-\lambda_e R(n+1)}) && (62)\\
&= (1 - e^{-\lambda_e R/2}) \left[\sum_{n=1}^{\infty} (1 - p_v^{2n+1})(e^{-\lambda_e R(n)}) + \sum_{n=1}^{\infty} (1 - p_v^{2n+2})(e^{-\lambda_e R/2(2n+1)}) \right] && (63)\\
&= (1 - e^{-\lambda_e R/2}) \left[\sum_{n=1}^{\infty} e^{-\lambda_e R n} - p_v \sum_{n=1}^{\infty} p_v^{2n} e^{-\lambda_e R n} \right. \\
&\quad \left. + e^{-\lambda_e R/2} \sum_{n=1}^{\infty} e^{-\lambda_e R n} - p_v^2 e^{-\lambda_e R/2} \sum_{n=1}^{\infty} p_v^{2n} e^{-\lambda_e R n} \right] && (64)\\
&= (1 - e^{-\lambda_e R/2}) \left[\frac{e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} - p_v \frac{p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} + e^{-\lambda_e R/2} \frac{e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} - p_v^2 e^{-\lambda_e R/2} \frac{p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} \right] && (65)\\
&= (1 - e^{-\lambda_e R/2}) \left[(1 + e^{-\lambda_e R/2}) \frac{e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} - (1 + p_v e^{-\lambda_e R/2}) p_v \frac{p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} \right] && (66)\\
&= \left[e^{-\lambda_e R} - \frac{p_v (p_v e^{-\lambda_e R/2})^2 (1 - e^{-\lambda_e R/2})}{1 - p_v e^{-\lambda_e R/2}} \right] && (67)
\end{aligned}$$

Equation (60) is derived by substituting from equation (58). Equation (61) is derived by adjusting the limits of integral in equation (60). By integrating and expressing as sum of an infinite series, we obtain (62).

Finally, the equation (67) is obtained by evaluating the sum of infinite series. This expression gives us the upper bound probability of nodes being disconnected in the *System v*.
The upper bound for the corresponding density function is evaluated as:

$$\overline{f_{X|\bar{C}}(x)} = \frac{f_X(x)\overline{P(\bar{C}|X=x)}}{P(\bar{C})} = \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})}\left(\lambda_e e^{-\lambda_e x}(1-(1-e^{-\lambda_w R/2})^{\lceil 2x/R \rceil})\right) & \text{if } x > R \end{cases} \tag{68}$$

Substituting $p_v = (1 - e^{-\lambda_w R/2})$, we get:

$$\overline{f_{X|\bar{C}}(x)} = \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})}\left(\lambda_e e^{-\lambda_e x}(1-p_v^{\lceil 2x/R \rceil})\right) & \text{if } x > R \end{cases} \tag{69}$$

As described earlier, the expected number of cells traversed until connectivity is achieved is analogous to the *pattern matching* problem described in Section 4.3. Thus, applying Equation (4) to find the upper bound for expected distance for a given separation $X_i = x$, cell size $R/2$, we obtain the following expression:

$$\overline{E[D_1|X=x]} = \left[\frac{1-p_v^{\lceil 2x/R \rceil}}{(1-p_v)p_v^{\lceil 2x/R \rceil}} - \left\lceil \frac{2x}{R} \right\rceil\right]\frac{R}{4} \tag{70}$$

Here, $p_v = (1 - e^{-\lambda_w R/2})$, the probability that consecutive *westbound* nodes are connected in the *System v*. The above expression in Equation (70) signifies the upper bound for the distance traversed until connectivity *westbound* traffic is achieved. This distance is computed by finding the upper bound for expected number of cells until connectivity and finding the corresponding distance, given the cell size is $R/2$. Note that a correction factor of $1/2$ is applied as nodes in either direction *westbound* and *eastbound* are traveling at speed vm/s. Thus, the required distance is effectively halved. Furthermore, the expression in Equation (4) gives us the cells until after the pattern is seen. The number of cells traversed until before the pattern is seen is the expected number of cells minus the desired number of cells. Hence, we subtract $\lceil 2x/R \rceil$ to find the upper bound distance until connectivity.

Generalizing the result in Equation (70) for exponential distribution of nodes:

$$\overline{E[D_1]_v} = \int_0^{\infty} \overline{E[D_1|X] f_{X|\bar{C}}(x)} dx \tag{71}$$

$$= \int_R^{\infty} \frac{E[D_1|X] f_X(x) P(\bar{C}|X=x)}{P(\bar{C})} dx \tag{72}$$

$$= \frac{1}{P(\bar{C})} \int_R^{\infty} \left[\left(\frac{1 - p_v^{\lceil 2x/R \rceil}}{(1-p_v) p_V^{\lceil 2x/R \rceil}} - \left\lceil \frac{2x}{R} \right\rceil \right) \frac{R}{4} \right] \lambda_e e^{-\lambda_e x} (1 - p_v^{\lceil 2x/R \rceil}) dx \tag{73}$$

$$= \frac{R}{4P(\bar{C})} \left[\int_R^{\infty} \frac{(1 - p_v^{\lceil 2x/R \rceil})^2}{(1-p_v) p_v^{\lceil 2x/R \rceil}} \lambda_e e^{-\lambda_e x} dx - \int_R^{\infty} \left\lceil \frac{2x}{R} \right\rceil \lambda_e e^{-\lambda_e x} (1 - p_v^{\lceil 2x/R \rceil}) dx \right] \tag{74}$$

$$= \frac{R}{4P(\bar{C})} \left[\frac{1}{1-p_v} \left[\sum_{n=1}^{\infty} \frac{(1-p_v^{2n+1})^2}{p_v^{2n+1}} \int_{nR}^{(2n+1)\frac{R}{2}} \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} \frac{(1-p_v^{2n+2})^2}{p_v^{2n+2}} \int_{(2n+1)\frac{R}{2}}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx \right] \right.$$
$$\left. - \left[\sum_{n=1}^{\infty} (2n+1)(1-p_v^{2n+1}) \int_{nR}^{(2n+1)\frac{R}{2}} \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} (2n+2)(1-p_v^{2n+2}) \int_{(2n+1)\frac{R}{2}}^{(n+1)R} \lambda_e e^{-\lambda_e x} dx \right] \right] \tag{75}$$

$$= \frac{R}{4P(\bar{C})} \left[\frac{1}{1-p_v} \left[\sum_{n=1}^{\infty} \frac{(1-p_v^{2n+1})^2}{p_v^{2n+1}} (e^{-\lambda_e nR} - e^{-\lambda_e (2n+1)\frac{R}{2}}) + \sum_{n=1}^{\infty} \frac{(1-p_v^{2n+2})^2}{p_v^{2n+2}} (e^{-\lambda_e (2n+1)\frac{R}{2}} \right. \right.$$
$$\left. - e^{-\lambda_e (n+1)R}) \right] - \left[\sum_{n=1}^{\infty} (2n+1)(1-p_v^{2n+1})(e^{-\lambda_e nR} - e^{-\lambda_e (n+1)R}) \right. \tag{76}$$

$$\left. \left. + \sum_{n=1}^{\infty} (2n+2)(1-p_v^{2n+2})^2 (e^{-\lambda_e (2n+1)\frac{R}{2}} - e^{-\lambda_e (n+1)R}) \right] \right] \tag{77}$$

$$= \frac{R(1 - e^{\lambda_e R/2})}{4P(\bar{C})} \left[\frac{1}{1-p_v} \left[\sum_{n=1}^{\infty} \frac{(1-p_v^{2n+1})^2}{p_v^{2n+1}} (e^{-\lambda_e nR}) + \sum_{n=1}^{\infty} \frac{(1-p_v^{2n+2})^2}{p_v^{2n+2}} (e^{-\lambda_e (2n+1)R/2}) \right] \right.$$
$$\left. - \left[\sum_{n=1}^{\infty} (2n+1)(1-p_v^{2n+1})(e^{-\lambda_e nR}) + \sum_{n=1}^{\infty} (2n+2)(1-p_v^{2n+2})^2 e^{-\lambda_e (2n+1)\frac{R}{2}} \right] \right] \tag{78}$$

$$= \frac{R(1 - e^{\lambda_e R/2})}{4P(\bar{C})} \left[\frac{1}{1-p_v} \left[\frac{e^{-\lambda_e R}}{p_v^2 - e^{-\lambda_e R}} \frac{1}{p_v} \left(1 + \frac{e^{-\lambda_e R/2}}{p_v} \right) + p_v \frac{p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} \left(1 + p_v e^{-\lambda_e R/2} \right) \right. \right.$$
$$\left. - \frac{2e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} (1 + e^{-\lambda_e R/2}) \right] - \left[\frac{e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} \left(1 + 2e^{-\lambda_e R/2} \right) + \frac{2e^{-\lambda_e R}}{(1 - e^{-\lambda_e R})^2} \left(1 + e^{-\lambda_e R/2} \right) \right.$$
$$\left. \left. - \frac{p_v p_v^2 e^{-\lambda_e R}}{(1 - p_v^2 e^{-\lambda_e R})} \left(1 + 2p_v e^{-\lambda_e R/2} \right) - p_v \frac{2p_v^2 e^{-\lambda_e R}}{(1 - p_v^2 e^{-\lambda_e R})^2} \left(1 + p_v e^{-\lambda_e R/2} \right) \right] \right] \tag{79}$$

Equation (72) is derived by substituting from Equation (**??**). Equation (75) is derived by adjusting the limits of integral in Equation (74). By integrating and expressing as sum of an infinite series, we obtain (77). Finally, the equation (79) is obtained by evaluating the sum of infinite series. This expression gives us the average distance of data propagation in Phase 1.

Phase2

Similar to the upper bound derivation, once connectivity is achieved, the gap between *eastbound* nodes, ($X_i > R$), is *bridged* by *westbound* nodes. However, the distance is covered at at multi-hop radio speed

v_{radio} m/s, so we compute the lower bound for the expected distance between *eastbound* nodes given that the nodes were disconnected. This distance is covered after Phase 1 once connectivity is achieved. To derive a lower bound limit, we compute the lower bound probability that nodes are disconnected.

$$\underline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1-(1-e^{-\lambda_w R/2})^{\lfloor 2x/R \rfloor} & \text{if } x > R \end{cases} \tag{80}$$

Substituting $p_v = (1-e^{-\lambda_w R/2})$:

$$\underline{P(\bar{C}|X=x)} = \begin{cases} 0 & \text{if } x \leq R, \\ 1-p_v^{\lfloor 2x/R \rfloor} & \text{if } x > R \end{cases} \tag{81}$$

The lower bound probability that nodes are disconnected for the *System v* ($\bar{C}$), independent of node distribution, is evaluated as:

$$\begin{aligned}
\underline{P(\bar{C})} &= \int_0^\infty \underline{P(\bar{C}|X=x)} f_X(x)dx && (82)\\
&= \int_R^\infty (1-p_v^{\lfloor 2x/R \rfloor})\lambda_e e^{-\lambda_e x}dx && (83)\\
&= \sum_{n=1}^{\infty}(1-p_v^{2n})\int_{nR}^{(2n+1)R/2}\lambda_e e^{-\lambda_e x}dx + \sum_{n=1}^{\infty}(1-p_v^{2n+1})\int_{(2n+1)R/2}^{(n+1)R}\lambda_e e^{-\lambda_e x}dx && (84)\\
&= \sum_{n=1}^{\infty}(1-p_v^{2n})(e^{-\lambda_e R(n)} - e^{-\lambda_e(2n+1)R/2}) + \sum_{n=1}^{\infty}(1-p_v^{2n+1})(e^{-\lambda_e(2n+1)R/2} - e^{-\lambda_e R(n+1)}) && (85)\\
&= (1-e^{-\lambda_e R/2})\left[\sum_{n=1}^{\infty}(1-p_v^{2n})(e^{-\lambda_e Rn}) + \sum_{n=1}^{\infty}(1-p_v^{2n+1})(e^{-\lambda_e(2n+1)R/2})\right] && (86)\\
&= (1-e^{-\lambda_e R/2})\left[\sum_{n=1}^{\infty}e^{-\lambda_e Rn} - \sum_{n=1}^{\infty}p_v^{n}e^{-\lambda_e Rn}e^{-\lambda_e R/2}\sum_{n=1}^{\infty}e^{-\lambda_e Rn} - p_v e^{-\lambda_e R/2}\sum_{n=1}^{\infty}p_v^{2n}e^{-\lambda_e Rn}\right] && (87)\\
&= (1-e^{-\lambda_e R/2})\left[\frac{e^{-\lambda_e R}}{1-e^{-\lambda_e R}}(1+e^{-\lambda_e R/2}) - \frac{p_v^2 e^{-\lambda_e R}}{1-p_v^2 e^{-\lambda_e R}}(1+p_v e^{-\lambda_e R/2})\right] && (88)\\
&= \left[e^{-\lambda_e R} - \frac{p_v^2 e^{-\lambda_e R}}{1-p_v e^{-\lambda_e R/2}}(1-e^{-\lambda_e R/2})\right] && (89)
\end{aligned}$$

Equation (83) is derived by substituting from equation (81). Equation (84) is derived by adjusting the limits of integral in equation (83). By integrating and expressing as sum of an infinite series, we obtain (85). Finally, the equation (89) is obtained by evaluating the sum of infinite series. This expression gives us the lower bound probability of nodes being disconnected in the *System v*.
The corresponding lower bound for the density function can be expressed as:

$$\begin{aligned}
\underline{f_{X|\bar{C}}(x)} &= \frac{f_X(x)\underline{P(\bar{C}|X=x)}}{\underline{\underline{P(\bar{C})}}} \\
&= \begin{cases} 0 & \text{if } x \leq R \\ \frac{1}{P(\bar{C})}\left(\lambda_e e^{-\lambda_e x}(1-(1-e^{-\lambda_w R/2})^{\lfloor 2x/R \rfloor})\right) & \text{if } x > R \end{cases}
\end{aligned}$$

Substituting $p_v = (1 - e^{-\lambda_w R/2})$, we get:

$$\underline{f_{X|\bar{C}}(x)} = \begin{cases} 0 & \text{if } x \leq R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - p_v^{\lfloor 2x/R \rfloor}) \right) & \text{if } x > R \end{cases} \tag{90}$$

Given the density function (Eq. (90)), the lower bound for average separation between *eastbound* nodes, given that they are not connected is evaluated as:

$$\underline{E[X|\bar{C}]} = \int_R^\infty x \underline{f_{X|\bar{C}}(x)} dx \tag{91}$$

$$= \int_R^\infty \frac{x f_X(x) \underline{P(\bar{C}|X = x)}}{\underline{P(\bar{C})}} dx \tag{92}$$

$$= \int_R^\infty \frac{x \lambda_e e^{-\lambda_e x} (1 - p_v^{\lfloor 2x/R \rfloor})}{P(\bar{C})} dx \tag{93}$$

$$= \frac{1}{P(\bar{C})} \left[\sum_{n=1}^{\infty} (1 - p_v^n) \int_{nR}^{(2n+1)R/2} x \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} (1 - p_v^{2n+1}) \int_{(2n+1)R/2}^{(n+1)R} x \lambda_e e^{-\lambda_e x} dx \right] \tag{94}$$

$$= \frac{1}{\lambda_e P(\bar{C})} \left[\sum_{n=1}^{\infty} (1 - p_v^n) \left[(1 + \lambda_e n R) e^{-\lambda_e n R} - (1 + \lambda_e (2n+1) R/2) e^{-\lambda_e (2n+1) R/2} \right] \right. \tag{95}$$

$$\left. + \sum_{n=1}^{\infty} (1 - p_v^{n+1}) \left[(1 + \lambda_e (2n+1) R/2) e^{-\lambda_e (2n+1) R/2} - (1 + \lambda_e (n+1) R) e^{-\lambda_e (n+1) R} \right] \right] \tag{96}$$

$$= \frac{1}{\lambda_e P(\bar{C})} \left[\sum_{n=1}^{\infty} (1 - p_v^n) (e^{-\lambda_e R n}) \left[1 + \lambda_e R(n) - (1 + \lambda_e R(n + 1/2)) e^{-\lambda_e R/2} \right] \right.$$

$$\left. + \sum_{n=1}^{\infty} (1 - p_v^{n+1}) (e^{-\lambda_e (2n+1) R/2}) \left[1 + \lambda_e (2n+1) R/2 - (1 + \lambda_e R(n+1)) e^{-\lambda_e R/2} \right] \right. \tag{97}$$

$$= \frac{1}{\lambda_e P(\bar{C})} \sum_{n=1}^{\infty} (1 - p_v^n) (e^{-\lambda_e R})^n \left[(1 - e^{-\lambda_e R/2})(1 + \lambda_e R n) - \lambda_e R/2 (e^{-\lambda_e R/2}) \right]$$

$$- \sum_{n=1}^{\infty} (1 - p_v^{n+1}) (e^{-\lambda_e R/2})^{2n+1} \left[(1 - e^{-\lambda_e R/2})(1 + \lambda_e R n) + \lambda_e R/2 - \lambda_e R (e^{-\lambda_e R/2}) \right] \tag{98}$$

$$= \frac{1}{\lambda_e P(\bar{C})} \left[e^{-\lambda_e R} + \frac{\lambda_e R e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} - \frac{1 - e^{-\lambda_e R/2}}{1 - p_v e^{-\lambda_e R/2}} \left[p_v^2 e^{-\lambda_e R} + \frac{\lambda_e R p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} \right] \right.$$

$$\left. - \lambda_e R e^{-\lambda_e R} \frac{e^{-\lambda_e R}}{1 - e^{-\lambda_e R}} + \lambda_e \frac{R}{2} e^{-\lambda_e R/2} \frac{p_v^2 e^{-\lambda_e R}}{1 - p_v^2 e^{-\lambda_e R}} (1 - p_v (1 - 2 e^{-\lambda_e R/2})) \right] \tag{99}$$

Equation (92) is derived by substituting from equation (90). Equation (93) is derived by adjusting the limits of integral in equation (92). By integrating and expressing as sum of an infinite series, we obtain (96). Finally, the equation (99) is obtained by evaluating the sum of infinite series. This expression gives us the lower bound of the distance covered once the connectivity is achieved in the *System v*.

In phase 2, the nodes are connected by multi-hop connectivity and the data are able to propagate multi-hop over connected nodes. As illustrated in Figure 9, the messages travel along *eastbound* nodes when connectivity along *eastbound* nodes is available. When the nodes *eastbound* are disconnected the messages are forwarded over nodes traveling in the *westbound* direction that are connected to the next *eastbound* node.

The lower bound probability of connectivity (C) for nodes for given value of inter-node distance ($X_i = x$) can be expressed as:

$$\underline{P(C|X=x)} = \begin{cases} 1 & \text{if } x \leq R, \\ (1-e^{-\lambda_w R/2})^{\lceil x/R \rceil} & \text{if } x > R \end{cases} \tag{100}$$

Substituting $p_v = (1 - e^{-\lambda_w R/2})$

$$\underline{P(C|X=x)} = \begin{cases} 1 & \text{if } x \leq R, \\ p_v^{\lceil x/R \rceil} & \text{if } x > R \end{cases} \tag{101}$$

The nodes are connected if the distance between nodes is within the communication range ($x \leq R$). In the event that the distance is beyond the communication range ($x > R$), connectivity over *westbound* traffic is sought. The distance corresponds to a maximum of $\lceil x/R \rceil$ nodes that separate the two *eastbound* nodes. The nodes along *eastbound* are connected if each of these cells is occupied by a node. The upper bound probability of this event is $((1 - e^{-\lambda_w R/2})^{\lceil 2x/R \rceil})$ Thus, the lower bound for the probability of connectivity for nodes along *eastbound* traffic $P(C)$, independent of inter-node distance X, is derived as:

$$\underline{P(C)} = \int_0^\infty \underline{P(C|X=x)} f_X(x) dx \tag{102}$$

$$= \int_0^R 1.\lambda_e e^{-\lambda_e x} dx + \int_R^\infty p_v^{\lceil 2x/R \rceil} \lambda_e e^{-\lambda_e x} dx \tag{103}$$

$$= \int_0^R \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} \int_{nR}^{(2n+1)R/2} p_v^{2n+1} \lambda_e e^{-\lambda_e x} dx + \sum_{n=1}^{\infty} \int_{(2n+1)R/2}^{(n+1)R} p_v^{2n+2} \lambda_e e^{-\lambda_e x} dx \tag{104}$$

$$= (1 - e^{-\lambda_e R}) + \sum_{n=0}^{\infty} p_v^{2n+1} (e^{-\lambda_e R})^n (1 - e^{-\lambda_e R/2}) + \sum_{n=0}^{\infty} p_v^{2(n+1)} (e^{-\lambda_e R/2})^{2n+1} (1 - e^{-\lambda_e R/2}) \tag{105}$$

$$= (1 - e^{-\lambda_e R}) + p_v (1 - e^{-\lambda_e R/2}) \frac{(p_v e^{-\lambda_e R/2})^2}{1 + p_v e^{-\lambda_e R/2}} \tag{106}$$

Equation (103) is derived by substituting from equation (101). Equation (104) is derived by adjusting the limits of integral in equation (103). By integrating and expressing as sum of an infinite series, we obtain Equation (105). Finally, the equation (106) is obtained by evaluating the sum of infinite series.

Given the connectivity, we can derive the physical distance of data propagation for a given distribution. The data propagation is conditional upon the density function of node distribution. Thus, the propagation distance is a function of connectivity along *eastbound* and *westbound* node and the density function. The upper bound for density function for data propagation distance is expressed as:

$$\begin{aligned} \underline{f_{X|C}(x)} &= \frac{\underline{\underline{f_X(x) P(C|X=x)}}}{\underline{P(C)}} \\ &= \begin{cases} \frac{1}{P(C)} \left(\lambda_e e^{-\lambda_e x} \right) & \text{if } x \leq R \\ \frac{1}{P(C)} \left(\lambda_e e^{-\lambda_e x} (1 - e^{-\lambda_w R/2})^{\lceil 2x/R \rceil} \right) & \text{if } x > R \end{cases} \end{aligned} \tag{107}$$

Substituting $p_v = (1 - e^{-\lambda_w R/2})$

$$
\begin{aligned}
\underline{f_{X|C}(x)} &= \underline{\underline{\frac{f_X(x)P(C|X=x)}{\underline{P(C)}}}} \\
&= \begin{cases} \frac{1}{P(C)}\left(\lambda_e e^{-\lambda_e x}\right) & \text{if } x \leq R \\ \frac{1}{P(C)}\left(\lambda_e e^{-\lambda_e x} p_v^{\lceil 2x/R \rceil}\right) & \text{if } x > R \end{cases}
\end{aligned} \tag{108}
$$

Thus, the lower bound for the expected distance of data propagation in Phase 2 is evaluated as:

$$
\underline{E[X|C]} = \int_0^\infty x \underline{f_{X|C}(x)} dx \tag{109}
$$

$$
= \int_0^\infty \frac{x f_X(x) P(C|X=x) dx}{P(C)} \tag{110}
$$

$$
= \frac{1}{P(C)} \left[\int_0^R \lambda_e x e^{-\lambda_e x} dx + \int_R^\infty \lambda_e x e^{-\lambda_e x} p_v^{\lceil 2x/R \rceil} dx \right] \tag{111}
$$

$$
= \frac{1}{P(C)} \left[\int_0^R \lambda_e x e^{-\lambda_e x} dx + \ldots + \int_{nR}^{(2n+1)R/2} \lambda_e x e^{-\lambda_e x} P_v^{2n+1} dx + \int_{(2n+1)R/2}^{(n+1)R} \lambda_e x e^{-\lambda_e x} p_v^{2(n+1)} dx \right] \tag{112}
$$

$$
= \frac{1}{P(C)} \left[\int_0^R \lambda_e e^{-\lambda_e x} x dx + \sum_{n=1}^\infty p_v^{2n+1} \int_{nR}^{(2n+1)R/2} \lambda_e x e^{-\lambda_e x} dx + \sum_{n=1}^\infty p_v^{2(n+1)} \int_{(2n+1)R/2}^{(n+1)R} \lambda_e x e^{-\lambda_e x} dx \right]
$$

$$
= \frac{1}{\lambda_e P(C)} \Bigg[[1-(1+\lambda_e R)e^{-\lambda_e R}] + \sum_{n=1}^\infty p_v^{2n+1} (e^{-\lambda_e R})^n [1+\lambda_e nR - (1+\lambda_e(2n+1)R/2)e^{-\lambda_e R/2}]
$$

$$
+ \sum_{n=1}^\infty p_v^{2n+1} (e^{-\lambda_e R/2})^{2n+1} [1+\lambda_e(2n+1)R/2 - (1+\lambda_e(n+1)R)e^{-\lambda_e R/2}] \Bigg] \tag{113}
$$

$$
= \frac{1}{\lambda_e P(C)} \Bigg[[1-(1+\lambda_e R)e^{-\lambda_e R}] + \sum_{n=1}^\infty p_v^{2n+1} (e^{-\lambda_e R})^n [(1+\lambda_e nR)(1-e^{-\lambda_e R/2}) - \lambda_e R/2 e^{-\lambda_e R/2}]
$$

$$
+ \sum_{n=1}^\infty p_v^{2n+1} (e^{-\lambda_e R/2})^{2n+1} [(1+\lambda_e nR)(1-e^{-\lambda_e R/2}) + \lambda_e R/2(1-2e^{-\lambda_e R/2})] \Bigg] \tag{114}
$$

$$
= \frac{1}{\lambda_e P(C)} \Bigg[[1-(1+\lambda_e R)e^{-\lambda_e R}] + p_v(1-e^{-\lambda_e R/2})(1+p_v e^{-\lambda_e R/2}) \left[\frac{p_v^2 e^{-\lambda_e R}}{1-p_v^2 e^{-\lambda_e R}} + \frac{\lambda_e R p_v^2 e^{-\lambda_e R}}{(1-p_v^2 e^{-\lambda_e R})^2} \right]
$$

$$
- p_v \lambda_e R/2 e^{-\lambda_e R/2} \frac{p_v^2 e^{-\lambda_e R}}{(1-p_v^2 e^{-\lambda_e R})} (1-p_v(1-2e^{-\lambda_e R/2}) \Bigg] \tag{115}
$$

Equation (110) is derived by substituting from equation (108). Equation (113) is derived by adjusting the limits of integral in equation (111). By integrating and expressing as sum of an infinite series, we obtain Equation (114). Finally, the equation (115) is obtained by evaluating the sum of infinite series. This expression gives us an lower bound for the distance of message propagation when there is multi-hop connectivity is available in Phase 2.

$$
\underline{E[D_2]_u} = \underline{\underline{\frac{E[X|C]P(C)}{1-\underline{P(C)}}}} \tag{116}
$$

$$
\tag{117}
$$

Approximation

In this section, we propose an approximation for the cell size. In the previous sections we considered an upper bound cell size of R and a lower bound for cell size $R/$. We now consider an approximation for the cell size = KR where $0 < K < 1$. We derive the expected distances covered in Phase 1 and Phase 2.

$$P(\bar{C}|X = x) = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - (1 - e^{-\lambda_w KR})^{x/KR} & \text{if } x > R \end{cases} \tag{118}$$

Substituting $p_a = (1 - e^{-\lambda_w KR})$:

$$P(\bar{C}|X = x) = \begin{cases} 0 & \text{if } x \leq R, \\ 1 - p_a^{x/KR} & \text{if } x > R \end{cases} \tag{119}$$

The lower bound probability that nodes are disconnected ($\bar{C}$), independent of node distribution, is evaluated as:

$$\begin{aligned} P(\bar{C}) &= \int_0^\infty P(\bar{C}|X = x) f_X(x) dx && (120) \\ &= \int_R^\infty (1 - p_a^{x/KR}) \lambda_e e^{-\lambda_e x} dx && (121) \\ &= \int_R^\infty \lambda_e e^{-\lambda_e x} dx - \int_R^\infty p_a^{x/KR} \lambda_e e^{-\lambda_e x} dx && (122) \\ &= \int_R^\infty \lambda_e e^{-\lambda_e x} dx - \int_R^\infty \lambda_e e^{-x(\lambda_e - 1/KR \ln p_a)} dx && (123) \\ &= e^{-\lambda_e R} - \left(\frac{\lambda_e (KR) e^{-\lambda_e R} p_a^{1/K}}{\lambda_e (KR) - \ln(p_a)} \right) && (124) \end{aligned}$$

The corresponding lower bound for the density function is evaluated as:

$$\begin{aligned} f_{X|\bar{C}}(x) &= \frac{f_X(x) P(\bar{C}|X = x)}{P(\bar{C})} \\ &= \begin{cases} 0 & \text{if } x \leq R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - (1 - e^{-\lambda_w KR})^{x/KR}) \right) & \text{if } x > R \end{cases} \end{aligned}$$

Substituting $p_a = (1 - e^{-\lambda_w KR})$, we get:

$$f_{X|\bar{C}}(x) = \begin{cases} 0 & \text{if } x \leq R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - p_a^{x/KR}) \right) & \text{if } x > R \end{cases} \tag{125}$$

$$E[D_1|X = x] = \left[\frac{1 - p^{x/KR}}{(1 - p) p^{x/KR}} - \frac{x}{KR} \right] \frac{KR}{2} \tag{126}$$

Generalizing the result in Equation (126) for the exponential distribution of nodes:

$$E[D_1]_a = \int_0^\infty E[D_1|X] f_{X|\bar{C}}(x) dx \tag{127}$$
$$= \int_R^\infty \frac{E[D_1|X] f_X(x) P(\bar{C}|X=x)}{P(\bar{C})} dx \tag{128}$$
$$= \frac{1}{P(\bar{C})} \int_R^\infty \left[\left(\frac{1 - p_a^{x/KR}}{(1-p_a) p_a^{x/KR}} - \frac{x}{KR} \right) \frac{KR}{2} \right] \lambda_e e^{-\lambda_e x} (1 - p_a^{x/KR}) dx \tag{129}$$
$$= \frac{KR}{2P(\bar{C})} \left[\int_R^\infty \frac{(1 - p_a^{x/KR})^2}{(1-p_a) p_a^{x/KR}} \lambda_e e^{-\lambda_e x} dx - \int_R^\infty \frac{x}{KR} \lambda_e e^{-\lambda_e x} (1 - p_a^{x/KR}) dx \right] \tag{130}$$
$$= \frac{KR}{2P(\bar{C})} \left[\frac{1}{1-p_a} \int_0^\infty \frac{(1 - p_a^{x/KR})^2}{p_a^{x/KR}} \lambda_e e^{-\lambda_e x} dx - \int_0^\infty \frac{x}{KR} (1 - p_a^{x/KR}) \lambda_e e^{-\lambda_e x} dx \right] \tag{131}$$
$$= \frac{KR}{2P(\bar{C})} \left[\frac{\lambda_e}{1-p_a} \left[\frac{e^{-\lambda_e R} p_a^{-1/K}}{\lambda_e + \frac{1}{KR} \ln(p_a)} + \frac{e^{-\lambda_e R} p_a^{1/K}}{\lambda_e - \frac{1}{KR} \ln(p_a)} - \frac{2e^{-\lambda_e R}}{\lambda_e} \right] \right.$$
$$\left. - \frac{\lambda_e}{KR} \left[\frac{(1+\lambda_e R) e^{-\lambda_e R}}{\lambda_e^2} - \frac{((\lambda_e - \frac{1}{KR} \ln(p_a)) R + 1) e^{-\lambda_e R} p_a^{1/K}}{(\lambda_e - \frac{1}{KR} \ln(p_a))^2} \right] \right] \tag{132}$$

The corresponding density function can be expressed as:

$$f_{X|\bar{C}}(x) = \frac{f_X(x) P(\bar{C}|X=x)}{P(\bar{C})}$$
$$= \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - (1 - e^{-\lambda_w RK})^{x/KR}) \right) & \text{if } x > R \end{cases}$$

Substituting $p_a = (1 - e^{-\lambda_w KR})$, we get:

$$\overline{f_{X|\bar{C}}(x)} = \begin{cases} 0 & \text{if } x \le R \\ \frac{1}{P(\bar{C})} \left(\lambda_e e^{-\lambda_e x} (1 - p_a^{x/KR}) \right) & \text{if } x > R \end{cases} \tag{133}$$

$$E[X|\bar{C}] = \int_R^\infty x f_{X|\bar{C}}(x) dx \tag{134}$$
$$= \int_R^\infty \frac{x f_X(x) P(\bar{C}|X=x)}{P(\bar{C})} dx \tag{135}$$
$$= \int_R^\infty \frac{x \lambda_e e^{-\lambda_e x} (1 - p_a^{x/KR})}{P(\bar{C})} dx \tag{136}$$
$$= \frac{1}{P(\bar{C})} \int_0^\infty x \lambda_e e^{-\lambda_e x} dx - \int (p_a^{x/KR}) x \lambda_e e^{-\lambda_e x} dx \tag{137}$$
$$= \frac{1}{\lambda_e P(\bar{C})} \left[\frac{e^{-\lambda_e R} (1 + \lambda_e R)}{\lambda_e} - \frac{\lambda_e ((\lambda_e R - \frac{1}{K} \ln(p_a)) + 1) e^{-\lambda_e R} p_a^{1/K}}{(\lambda_e - \frac{1}{KR} \ln(p_a))^2} \right] \tag{138}$$

$$P(C|X=x) = \begin{cases} 1 & \text{if } x \le R, \\ (1 - e^{-\lambda_w KR})^{x/KR} & \text{if } x > R \end{cases} \tag{139}$$

Substituting $p_a = (1 - e^{-\lambda_w KR})$, we get:

$$P(C|X = x) = \begin{cases} 1 & \text{if } x \leq R, \\ p_a^{x/KR} & \text{if } x > R \end{cases} \tag{140}$$

$$\begin{aligned} P(C) &= \int_0^\infty P(C|X = x) f_X(x) dx & (141) \\ &= \int_0^R 1.\lambda_e e^{-\lambda_e x} dx + \int_R^\infty p_a^{x/KR} \lambda_e e^{-\lambda_e x} dx & (142) \\ &= \int_0^R \lambda_e e^{-\lambda_e x} dx + \int_0^\infty \lambda_e e^{-x(\lambda_e - 1/KR \ln(p_a))} dx & (143) \\ &= (1 - e^{-\lambda_e R}) + \frac{\lambda_e e^{-\lambda_e R} p_a^{1/K}}{\lambda_e - \frac{1}{KR}\ln(p_a)} & (144) \end{aligned}$$

$$\begin{aligned} E[X|C] &= \int_0^\infty x f_{X|C}(x) dx & (145) \\ &= \int_0^\infty \frac{x f_X(x) P(C|X = x) dx}{P(C)} & (146) \\ &= \frac{1}{P(C)} \left[\int_0^R \lambda_e x e^{-\lambda_e x} dx + \int_R^\infty \lambda_e x e^{-\lambda_e x} p_a^{x/KR} dx \right] & (147) \\ &= \frac{1}{P(C)} \left[\int_0^R \lambda_e e^{-\lambda_e x} x dx + \int_R^\infty \lambda_e e^{-x(\lambda_e - 1/KR \ln(p_a))} x dx \right] & (148) \\ &= \frac{1}{P(C)} \left[\frac{1 - (1 + \lambda_e R) e^{-\lambda_e R}}{\lambda_e} + \frac{(\lambda_e R - \frac{1}{K}\ln(p_a)) + 1) e^{-\lambda_e R} p_a^{1/K}}{(\lambda_e - \frac{1}{KR}\ln(p_a))^2} \right] & (149) \end{aligned}$$

$$\begin{aligned} E[D_2]_a &= \frac{E[X|C]P(C)}{1 - P(C)} & (150) \\ & & (151) \end{aligned}$$

5 Performance Results

We evaluate the performance of the messaging scheme with the help of the analytical model. Simultaneously, we simulate the VANET environment by generating vehicle traffic on either side of the roadway and studying the performance of messaging in the simulated network. We compare the results obtained from the analytical model with those obtained from the simulation. The results essentially demonstrate the performance of the messaging scheme in a network with time-varying connectivity as a function of the vehicle traffic density, transmission range and vehicle speed.

For comparison, we chose parameters for message propagation speed as $v_{radio} = 1000$ m/s. The radio range is $R = 125$ m and the vehicles speed is assumed to be $v = 20$ m/s (72kph/45mph). The traffic density is varied from over a range of 1 vehicle/km to 100 vehicles/km, to cover the low, intermediate and high traffic density scenarios.

Effective Message Propagation Rate

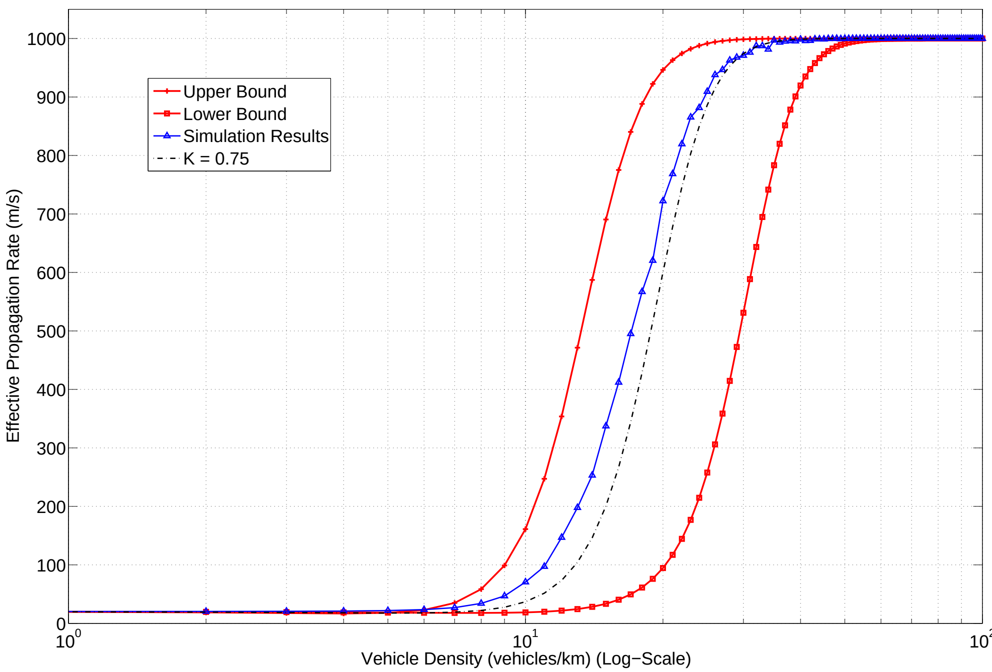


Figure 10: Simulation results – Comparison of effective propagation rates for Simulation and Analytical Results Message Propagation rate as density in the network increases.

Results in Figure 10 depict the effective message propagation rate for increasing vehicular traffic density. The traffic density is assumed to be numerically equivalent in both *eastbound* and *westbound* direction. This is for ease of representation and the parameters can be separately modified as evident from the analysis in Section 4. We compare the analytical *upper* bound and *lower* bound derived in Equations **??** and **??**, respectively, with the simulated results. As explained previously the message propagation essentially occurs in two alternating phases. For a given network density, the network experiences transient network connectivity as the messages propagate a physical distance. While multi-hop connectivity is available, messages are able to propagate at multi-hop radio speed (v_{radio} m/s). When the network is disconnected, the messages propagate at vehicle speed (v m/s), where $v_{radio} >> v$. Thus, the effective message propagation rate is a function of the time spent in each phase.

In Figure 10, we plot the *upper* bound, *lower* bound and the approximation results derived in . It is evident that when the mean value of vehicle traffic density is below 10 vehicles/km, the network is largely disconnected and the messages are buffered within vehicles. The data traverse a physical distance at vehicle speed ($v = 20$ m/s). When the node density is high (> 50 vehicles/km), the network is largely connected. Thus, data are able to propagate multi-hop through the network at the maximum speed permitted by the radio ($v_{radio} = 1000$ m/s). In medium node density, the network is comprised of disconnected sub-nets. There is transient connectivity in the network as vehicular traffic moves in opposing directions. As a result of the delay tolerant networking assumption and opportunistic forwarding, the message propagation alternates in the two phases. The effective rate, a function of the time spent in each phase, is between the two extremes of v m/s and v_{radio} m/s. Thus, the message propagation rate is a function of the connectivity in the network that is in turn determined by the vehicular traffic density for constant transmission range.

For the *upper* bound, the connectivity requirement is a necessary condition but not necessarily sufficient. Thus, the *upper* bound considers an optimistic approach to connectivity and hence, the curve leads the performance for a particular density. While for the *lower* bound, the connectivity is a sufficient condition but not necessary. Thus, the requirement is of more nodes than actually needed and hence, the performance lags for the same density. The simulation results show the performance of messaging in a simulated network environment. The results are averaged over several iterations to account for the random node generation and the resulting topology. The simulation results lie well within the *upper* and the *lower* bounds. The approximation derived in Section 4.5 closely follows the simulation results. Thus, we are able to demonstrate that the analytical model captures the essence of messaging in the VANET environment characterized by time varying connectivity and delay tolerant networking assumption.

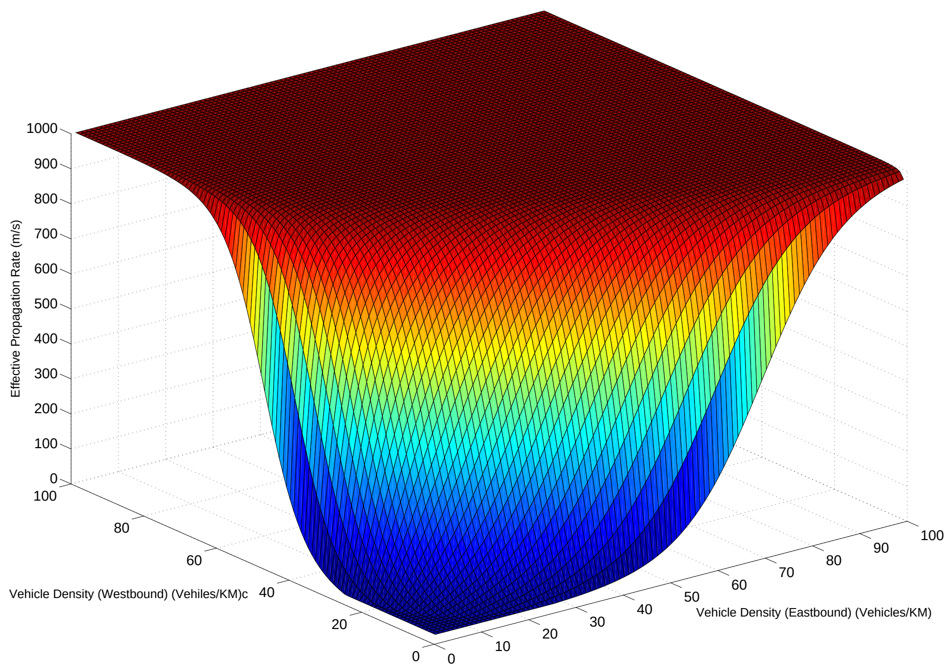


Figure 11: Simulation results – Results for Message Propagation Rate as a function of various *Eastbound* and *Westbound* vehicular traffic densities.

In Figure 11, we relax the assumption of symmetric values of traffic density along *eastbound* and *westbound* directions. We plot the performance of the messaging based on the approximation developed in Section 4.5 for values of *eastbound* and *westbound* traffic ranging from 1 vehicle/km to 100 vehicles/km. As is evident from the graph, the message rate increases as a function of the vehicular traffic density on either side of the roadway. The 3-dimensional graph allows us to map the performance of the messaging for asymmetric values of traffic density on either side of the roadway. For example, if both the *eastbound* and *westbound* directions have low traffic density of ~ 10 vehicles/km, then the node density is insufficient to enable message propagation. However, if the node density in the *eastbound* roadway low, ~ 20 vehicles/km, while the *westbound* direction has higher traffic density of 40 vehicles/km, then the node density is sufficient to achieve the maximum performance of 1000 m/s.

Phase Transition

It is interesting to note the *phase transition* of the message propagation rate from a minimum speed of $v = 20$ m/s to a maximum of $v_{radio} = 1000$ m/s as the node density of the network increases. The transition occurs as the network evolves with increasing vehicular traffic density from a sparse network, that is disconnected, to a dense network that is mostly fully connected. However, the transition is not sharp or pronounced owing to the unique nature of the messaging scheme that exploits the transient connectivity to enable data exchange. The scheme enables message propagation even when the network is not fully connected and hence, achieves gains in messaging performance.

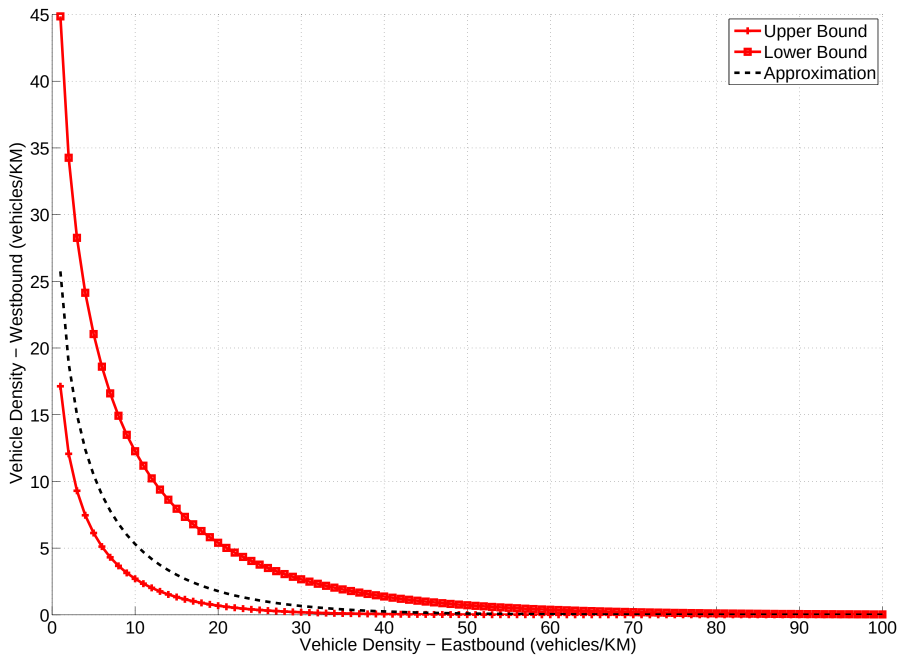


Figure 12: Simulation results – Minimum Density Relationship between *Eastbound* and *Westbound* vehicular traffic.

The analytical bounds enable us to evaluate the minimum density requirements for traffic in either direction of the roadway. The delay tolerant networking assumption is valid only when there is sufficient traffic density in the opposing direction to bridge partitions. This condition or requirement is evaluated in (**??**). The relationship between the *eastbound* and *westbound* traffic densities is shown in Figure 12. The figure shows that for low traffic density in the *eastbound* direction (< 10 vehicles/km), a relatively high density of traffic in the *westbound* direction, ($10-25$ vehicles/km) is required. While this result is intuitive, the mathematical relationship is only derived from the analytical model. This result is not evident from the simulation results due to the randomized generation of exponentially distributed vehicular traffic.

Critical Density of Phase Transition

In Figure 13, we compare the approximation model for the effective propagation rate with the performance of a MANET scheme such as AODV or DSR for a fixed source-destination separation of 12.5 kms. The MANET schemes rely on path formation and require end-to-end connectivity between the source-destination pairs. Thus, as a result, the scheme requires a high density of nodes for achieving end-to-end connectivity. It is evident that a scheme that utilises only one direction of traffic for connectivity requires a density of ~ 90 vehicles/km, on average. However, if vehicular nodes traveling in either direction are used for path formation, maximum performance is achieved at ~ 45 vehicles/km. Whereas, for the DTN assumption, the messaging performance is independent of the separation between the source-destination pairs, and is primarily a function of the vehicle density.

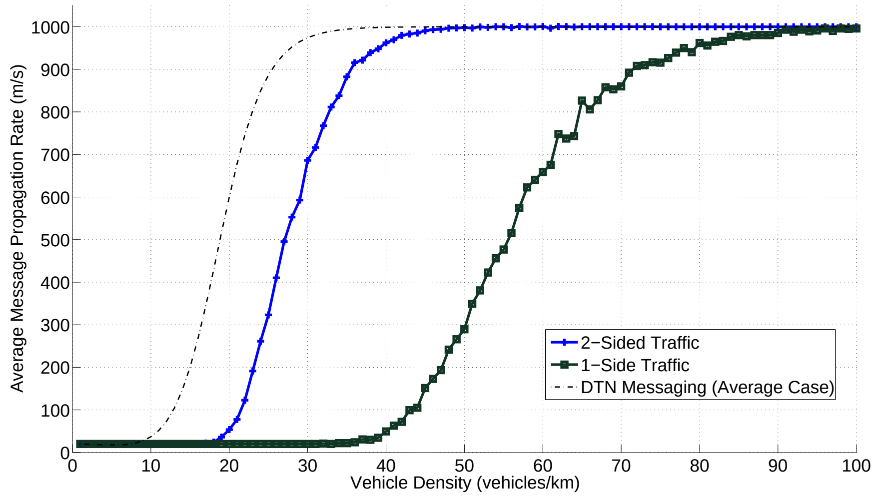


Figure 13: Simulation results – Comparison of DTN Messaging Strategy with a path formation based scheme utilizing 1-sided traffic and both sides of traffic for a distance of 12.5Kms.

Effect of Increased Mobility

In Figure 14, we observe the performance of the messaging scheme as the vehicular speed increases at fixed values of *eastbound* and *westbound* traffic density. The graph shows that the messaging performance increased by order of magnitude from 0 m/s to 200 m/s as vehicular mobility increases from 0 m/s to 20 m/s. This is counter-intuitive to the observation in conventional MANET protocols that increased mobility decreases the messaging performance owing to short-lived paths. However, in this connection-less messaging paradigm, it is observed that messaging performance is aided by increased mobility. The partitions that occur in the network are bridged at a faster rate leading to increased messaging performance.

6 Conclusion

In this work, we describe an analytical model for characterizing the behavior of message propagation in a VANET routing scheme as a function of traffic density on either side of the roadway and the physical radio characteristics. We present a unique routing scheme that assumes labelled data and delay tolerant networking to disseminate messages away from the point of interest. The simulation results show that the analytical model reflects the behavior of the routing scheme. We have characterized the behavior of message dissemination in a delay tolerant environment as a function of vehicular traffic density, vehicular speed and radio range. The analytical model reveals the relationship between the network parameters of radio range and vehicular traffic density that are not evident from simulation results, specifically the lower limit bound for convergence of delay tolerant network assumption. Of particular value is the observation that by increasing vehicular mobility we can actually improve message propagation, contrary to expectation, due to the increased interaction between fragments in the VANET. This result supports the argument for the use of delay tolerant and ad hoc routing techniques in future vehicular networks.

The model presented in this work is a design guide towards determining parameters for network setup or reconfigurable applications as varying traffic conditions demand. The model can be used to determine message dissemination delays for given vehicular traffic conditions and radio characteristics. Furthermore, it can be adapted to study the deployment of road-side infrastructure to support inter-vehicle communication.

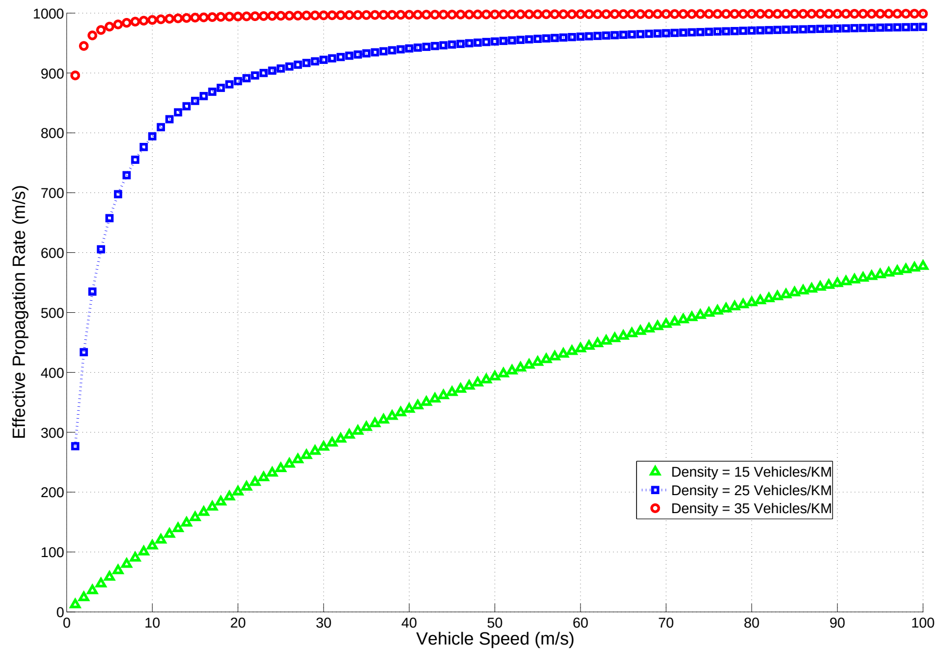


Figure 14: Simulation results – Comparison of impact of increasing density on effective propagation rate for various values of vehicular speed.

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